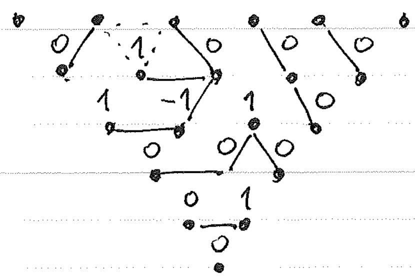
C.P.S. 1/23/2015

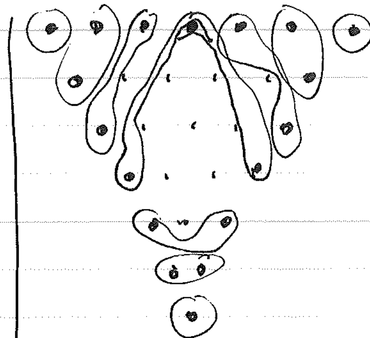
M. Glick

A problem from a paper of G. Carroll & D. Speyer 2004
on the cube recurrence & groves

Groves



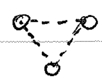
(n=5)



(n=6)

A grove of order n is a ~~subgraph~~ spanning forest of a triangle graph having n edges on each side such that

- Each component intersects the boundary
- The boundary is partitioned 3-fold symmetrically as shown here

Given a grove G, label each  by ~~2~~ $1-e$

where $e = \#$ of edges in G

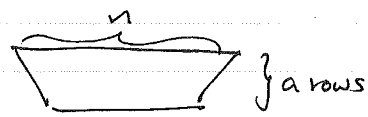
Call such an array of numbers $\{0,+1,-1\}$ an alternating sign triangle (AST) (without the grove).

PROBLEM: Give a direct (grove-free) definition of AST's.

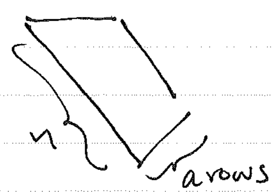
DEF'N: A permutation triangle (PT) is an AST with no -1's.

n	Groves(n) $\underset{\text{Carroll–Speyer}}{=} 3^{\lfloor n^2/4 \rfloor}$	AST(n)	PT(n)
1	1	1	1
2	3	3	3
3	9	6	6
4	81	35	27
5	729	151	91

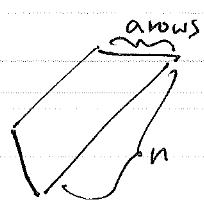
<u>CONJECTURE:</u> Let T be a size n triangular array with $\lfloor n/2 \rfloor$ 1's and the rest 0's. Then T is a PT iff

- 

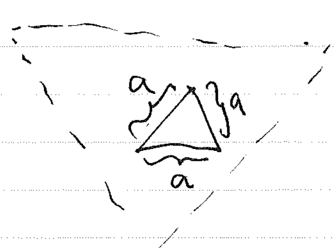
has at most a 1's for all a

- 

same story

- 

same story

- 

same story (e.g.

$$\begin{matrix} & 1 & \\ 1 & & 1 \end{matrix}$$

is impossible)