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Standard tableaux of shifted staircase shape

YT(n)

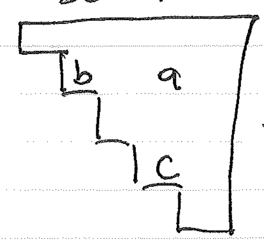
$$\overbrace{\begin{array}{cccc} 1 & 2 & 3 & 5 \\ & 4 & 6 & 7 \\ & & 8 & 9 \\ & & & 10 \end{array}}^{n=4}$$

Selberg tableaux :

ST(n)

$$\overbrace{\begin{array}{cccc} 1 & 3 & 2 & 6 \\ & 4 & 5 & 7 \\ & & 8 & 9 \\ & & & 10 \end{array}}^{n=4}$$

DEF'N:



$\Rightarrow b < a < c$

THM: $\overset{\text{(FACT 1)}}{ST(n)} = YT(n) \cdot 1!\, 2! \cdots (n-1)!$

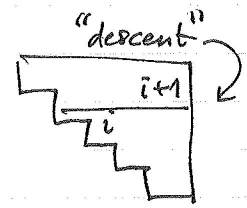
In fact, $\overset{\text{(FACT 2)}}{ST}(n; \underbrace{a_1, a_2, \ldots, a_n}_{\text{diagonal entries}}) = YT(n; \underbrace{a_1, a_2, \ldots, a_n}_{\text{same}}) \cdot 1!\, 2! \cdots (n-1)!$

PROBLEM 1: Find a bijective proof.

PROBLEM 2: Find a q-analogue.

One solution of q-FACT 1

For YT T, $\text{maj}(T) = \sum_{i \in \underbrace{\text{Des}(T)}_{\text{descents of } T}} i$

e.g. $T = \begin{array}{cccc} 1 & 2 & 3 & 5 \\ & 4 & 6 & 7 \\ & & 8 & 9 \\ & & & 10 \end{array}$ 

For ST T, $\text{maj}(T) := \sum_{i \in \text{Des}(T)} i$

where i is a descent if $i, i+1$ are out of order when T is read in Japanese order

$T = \begin{array}{cccc} 1 & 3 & 2 & 6 \\ & 4 & 5 & 7 \\ & & 8 & 9 \\ & & & 10 \end{array}$ Japanese rw 67910258341

<u>THM</u>: $$\sum_{T \in ST(n)} q^{\text{maj}(T)} = \sum_{T \in YT(n)} q^{\text{maj}(T)} \cdot q^N [1]!_q [2]!_q \cdots [n-1]!_q$$

$$N = \sum_{i=1}^{n} i(n-i)$$

<u>EXAMPLE</u>: $n=3$

YT(3) $\begin{array}{ccc} 1 & 2 & 3 \\ & 4 & 5 \\ & & 6 \end{array}$ $\begin{array}{ccc} 1 & 2 & 4 \\ & 3 & 5 \\ & & 6 \end{array}$

q^0 q^3

descents

$$\begin{array}{ccc} 1 & 2 & 3 \\ & 4 & 5 \\ & & 6 \end{array} \qquad 1+2+4 = 7 \rightsquigarrow q^7$$

$$\begin{array}{ccc} 1 & 2 & 4 \\ & 3 & 5 \\ & & 6 \end{array} \qquad 1+3=4 \rightsquigarrow q^4$$

$$\begin{array}{ccc} 1 & 3 & 2 \\ & 4 & 5 \\ & & 6 \end{array} \qquad 1+4=5 \rightsquigarrow q^5$$

$$\begin{array}{ccc} 1 & 2 & 5 \\ & 3 & 4 \\ & & 6 \end{array} \qquad 1+3+4 \rightsquigarrow q^8$$

$$\sum_{T \in ST(3)} q^{maj(T)} \overset{?}{=} \sum_{T \in YT(3)} q^{maj(T)} \cdot q^4 [1]_q [2]_q$$

$$\sum_{T \in ST(3)} q^{maj(T)} = q^4+q^5+q^7+q^8 = q^4(1+q)(1+q^3)$$

$$\sum_{T \in YT(3)} q^{maj(T)} \cdot q^4 [1]_q [2]_q = (1+q^3) q^4 (1+q)$$