Math 4242 FINAL EXAMINATION Friday, December 20, 2002.
Professor Peter A. Rejto.
Ten problems, each problem worth 40 points,

1. State and prove the Schwarz inequality.

2. Find the projection of the vector $b = (0, 3, 0)$ onto the line of the vector $a = (2, 2, -1)$. In other words, find a scalar γ such that the vector $b - \gamma a$ is orthogonal to a. (Then the projection is γa.)

3. Let the matrix A be given by,
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -6 & -2 \\ -2 & 7 & 5 \end{bmatrix}.$$
   (a) (20 pts.) Find an orthonormal basis for the column space of A
   (b) (20 pts.) Next, let the vector b be given by,
$$b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$
   Find the orthogonal projection of this vector b onto $col(A)$, the column space of A

4. Let the matrix A of the previous problem be replaced by the matrix:
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$
   Then find the projection of the vector b of the previous problem onto the column space of this matrix.

5. Let the function $y(x)$ be equal to 1 for $-\pi < x < 0$ and equal to -1 for $0 < x < \pi$. Find the first three Fourier coefficients of the function y. In other words, write
$$y(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + \ldots \qquad (1)$$
   Then find b_6.

6. Let A be a given matrix and let λ be a given number. Define that λ is an eigenvalue of A.

7. Find the eigenvalues and eigenvectors of the matrix:
$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

8. Let A be a given matrix and let λ_1 and λ_2 be two eigenvalues of A such that $\lambda_1 \neq \lambda_2$. Prove that the corresponding eigenvectors are linearly independent.

9. Diagonalize the matrix A of Problem 7. That is to say, find an invertible matrix S and a diagonal matrix Λ such that

$$A = S \Lambda S^{-1}$$

10. Let A denote the matrix of Problem 7. Show that every solution of the system of differential equations,

$$\frac{dx}{dt}(t) = Ax(t),$$

is bounded in the variable t.

GOOD LUCK