

**Math 4242            Test 1            Friday, September 27, 2002.**  
**Professor Peter A. Rejto.**  
**Five problems, each problem worth 30 points,**

1. Either, find all the solutions to the following system of linear equations, or show that they do not exist:

$$\begin{array}{rclcl} 2x & +y & +z & = & 5 \\ 4x & -6y & & = & -2 \\ -2x & +7y & +2z & = & 9 \end{array}$$

2. First, find the inverse of the following matrix, if it does exist.

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$$

Second, show that the matrix you have found, is indeed the inverse, or show that the inverse does not exist.

3. Let the elementary matrix  $E$  be given by,

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

First, find the inverse of  $E$ . Second, show that the matrix you have found, is indeed the inverse of  $E$ .

4. Let  $A$  be a given  $3 \times 3$  matrix and suppose that  $\det A \neq 0$ . Prove that  $A^{-1}$  does exist.

5. Let  $A$  be a given  $3 \times 3$  matrix and suppose that  $A^{-1}$  exists. Prove that  $\det A \neq 0$ .