Math 4242 Test 2 Friday, October 25, 2002.
Professor Peter A. Rejto.
Five problems, each problem worth 30 points,

1. Give an example of a vector space and three linearly independent vectors in it.

2. (a) Let V be an abstract vector space and let $v_1, v_2 and v_3$ be vectors in V. Define that these vectors are linearly independent.
   (b) Define that these three vectors span V.
   (c) Define that these three vectors form a basis for V.

3. Let the matrix A be given by,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}.$$

   (a) Find the dimension of the space of column vectors of A.
   (b) Find the dimension of the space of raw vectors of A.
   (c) (This part is independent of the matrix A.) Recall that all $3x3$ matrices form a vector space with respect to componentwise addition and componentwise multiplication by a scalar. Find the dimension of this vector space.

4. Let V be a vector space and let b_1 be a set of basis vectors consisting of the single element b_1. Next let $c_1 ... c_m$ be another set of basis vectors. Prove that $m = 1$. Hint: First show that it is no loss of generality to assume that $m = 2$.

5. Finally a mathematical pathology from our text. We define a "goofy" addition on R^2 by:

$$(a_1, a_2) +_g (b_1, b_2) = (a_1 + b_1 + 1, a_2 + b_2 + 1)$$

and keep the definition of multiplication by a scalar unchanged. Prove that R^2 is not a vector space with respect to the addition $+_g$.

GOOD LUCK