Math 4242 Test 3 Friday, November 22, 2002.
Professor Peter A. Rejto.
Five problems, each problem worth 30 points,

1. (a) (15 pts.) Give an example of an orthonormal basis in R^2.

   (b) (15 pts.) Expand the vector $v = (5, 6)$ with respect to your orthonormal basis of part (a). In other words, let e_1, e_2 denote the orthonormal basis that you have found in part (a). Next write

$$(5, 6) = \alpha_1 e_1 + \alpha_2 e_2. \tag{1}$$

   Then find α_1 and α_2

2. Find the projection of the vector $b = (0, 3, 0)$ onto the line of the vector $a = (2, 2, -1)$. In other words, find a scalar γ such that the vector $b - \gamma a$ is orthogonal to a. (Then the projection is γa.)

3. Let the matrix A be given by,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -6 & -2 \\ -2 & 7 & 5 \end{bmatrix}.$$

   (a) (15 pts.) Find a basis for the column space of A

   (b) (15 pts.) Find an orthonormal basis for the column space of A

4. Let A be a given 3×3 matrix. Prove that the column space of A is orthogonal to the nullspace of A. (You might recall that this is a special case of the Second Fundamental Theorem of Linear Algebra of our text.)

5. Let the function $y(x)$ be equal to 1 for $-\pi < x < 0$ and equal to -1 for $0 < x < \pi$. Find the first three Fourier coefficients of the function y. In other words, write

$$y(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos x + ... \tag{2}$$

   Then find a_0, a_1 and b_1.

GOOD LUCK