Math 4606 Test 1 March, 19, 2001.

Professor Peter A. Rejto

Name (Print): ______________________ Student ID number:__________

Section number: __________________ Name of TA: __________________

Signature:________________________

Additional Information:

The sequence $\{a_n\}$ is Cauchy if to every $\epsilon > 0$ there is number N such that for every number n and m:

$$n > N \text{ and } m > N \text{ implies } |a_n - a_m| \leq \epsilon.$$

Name (Print): ______________________ Student ID number: __________

1. (20 pts.) (Bolzano – Weierstrass Theorem in R^1)
   Let the sequence $\{a_n\} \in R^1$ be bounded in the sense that

$$\sup_{n<\infty} |a_n| < \infty.$$

Prove that $\{a_n\} \in R^1$ has a convergent subsequence.

Name (Print): ______________________ Student ID number: ______

2. (20 pts.) Let the function f mapping R^2 into R^1 be defined by,

$$f(x, y) = xy.$$

Prove that f is continuous at each point (x, y) of R^2.

Name (Print): ______________________ Student ID number: ______

3. (20 pts.) Let f be the function of the previous Problem and for a given R let the closed disk $D(0, R)$ be defined by,

$$D(0, R) = \{(x, y) : x^2 + y^2 \leq R^2\}.$$

Prove that for each R the function f is uniformly continuous on the closed disk $D(0, R)$.

Name (Print): ______________________ Student ID number: ______

4. (20 pts.) Let the function f map the set S into R^n. Suppose that the set S is sequentially compact and that f is continuous. Prove that f is uniformly continuous on S.

Name (Print): ____________________ Student ID number: ______

5. (20 pts.) Let the function f map the set S into R^1. Suppose that the set S is sequentially compact and that f is continuous. Prove that f is bounded on S.