**Math 4606 Test 3 April, 18, 2001.**

**Professor Peter A. Rejto**

Name (Print): ______________________ Student ID number: ______________
Section number: ____________________ Name of TA: ____________________
Signature: ________________________

**Additional Information:**

The sequence $\{a_n\}$ is Cauchy if to every $\epsilon > 0$ there is number $N$ such that for every number $n$ and $m$:

$$n > N \text{ and } m > N \text{ implies } |a_n - a_m| \leq \epsilon.$$

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(1) (25 pts.) Let the function $f$ mapping $R^1$ into $R^1$ be differentiable at the point $x \in R^1$ and let $f'(x)$ denote its derivative. Prove that

$$\lim_{|h| \to 0} \frac{|f(x+h) - f(x) - f'(x)h|}{|h|} = 0.$$

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(2) (25 pts.) Let the function $f$ map $R^2$ into $R^1$. Define that $f'(x)$ is a derivative of the function $f$ at the point $x \in R^2$.

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(3) (25 pts.) State and sketch the proof of the Mean Value theorem.

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(4) (25 pts.) Suppose the function $f$ mapping $R^2$ into $R^1$ has continuous partial derivatives and at the point $x = (x_1, x_2) \in R^2$ define

$$Df(x) = (\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}).$$

Prove that $Df(x)$ is a derivative of the funtion $f$ at the point $x \in R^2$.