HA2 – some solutions

1.5.24 (g)

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 4 & 2 & 3 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{(2)\to(2)-2*(1)} \left(\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{(2)\leftrightarrow(3)} \left(\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right) \xrightarrow{\text{diag}} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right)$$

$$\xrightarrow[(1)\to(1)-(3)]{(2)\to(2)+(3)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right)$$

inverse

1.6.19 $$(A^2)^T = (A \cdot A)^T \underset{(1.53)}{=} A^T A^T = (A^T)^2$$

$$\underset{A\ \text{symm}}{=} A^2 \implies A^2 \text{ symmetric}$$

1.6.24 $\underset{(1.53)}{(A^TA)^T = A^TA}$ ✓ $K^T = K$

1.8.1. (c)

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 2 & -1 & 3 & 7 \\ 1 & -2 & 5 & 1 \end{array}\right] \rightsquigarrow \begin{array}{cccc} 1 & 1 & -2 & -3 \\ 0 & -3 & +7 & 13 \\ 0 & -3 & 7 & 4 \end{array}$$

$$\rightsquigarrow \begin{array}{cccc} 1 & 1 & -2 & -3 \\ 0 & -3 & 7 & 13 \\ 0 & 0 & 0 & -9 \end{array}$$

$0 = -9$, <u>no sol'</u>

(d) $$\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 2 & 1 & -3 & -3 \\ 1 & -3 & 3 & 10 \end{array} \rightsquigarrow \begin{array}{cccc} 1 & -2 & 1 & 6 \\ 0 & 5 & -5 & -15 \\ 0 & -1 & 2 & 4 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 5 & -5 & -15 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z = 1$, $y = -2$, $x = 1$

<u>unique sol)</u>

(e) $$\left[\begin{array}{cccc|c} 1 & -2 & 2 & -1 & 3 \\ 3 & 1 & 6 & 11 & 16 \\ 2 & -1 & 4 & 1 & 9 \end{array}\right] \rightsquigarrow \begin{array}{ccccc} 1 & -2 & 2 & -1 & 3 \\ 0 & 7 & 0 & 14 & 7 \\ 0 & 3 & 0 & 3 & 3 \end{array}$$

$$\rightsquigarrow \begin{array}{ccccc} 1 & -2 & 2 & -1 & 3 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \rightsquigarrow$$

$w = 0$
$z = s$ arb.
$y = 1$
$x = 5 - 2s$

∞-many

1.8.7 (h)

$$\begin{matrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 2 & -3 & -1 \\ 4 & -1 & 3 & 2 \\ 0 & 3 & -5 & -2 \end{matrix} \rightsquigarrow \begin{matrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -5 & -2 \\ 0 & 3 & -5 & -2 \\ 0 & 3 & -5 & -2 \\ 0 & 3 & -5 & -2 \end{matrix} \rightsquigarrow \begin{matrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -5 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

rank 2

1.8.17 → row operations give

$$\begin{matrix} 1 & 2 & 3 & \dots & n \\ n & n & n & \dots & n \\ 1 & 2 & 3 & \dots & n \\ 1 & 2 & 3 & \dots & n \\ & \vdots \end{matrix} \rightsquigarrow \begin{matrix} 1 & 2 & 3 & \dots & n \\ 1 & 1 & 1 & \dots & 1 \\ 0 & & \dots & & 0 \\ 0 & & \dots & & 0 \end{matrix}$$

$$\rightsquigarrow \begin{matrix} 1 & 2 & 3 & \dots & n \\ 0 & 1 & 2 & 3 \dots & n \\ 0 & & \dots & & 0 \\ 0 & & \dots & & 0 \end{matrix} \Big\} \text{ rank } 2$$

1.9.1 (d)

$$\begin{matrix} 0 & 1 & -1 \\ -2 & 1 & 3 \\ 2 & 7 & -8 \end{matrix} \xrightarrow{1\leftrightarrow 3} \begin{matrix} 2 & 7 & -8 \\ -2 & 1 & 3 \\ 0 & 1 & -1 \end{matrix} \rightsquigarrow \begin{matrix} 2 & 7 & -8 \\ 0 & 8 & -5 \\ 0 & 1 & -1 \end{matrix}$$

$$\rightsquigarrow \begin{matrix} 2 & 7 & -8 \\ 0 & 8 & -5 \\ 0 & 0 & -3/8 \end{matrix} \rightsquigarrow \det = (-1)^1 \cdot 2 \cdot 8 \cdot (-3/8) = 6$$

1.9.7 : product formula