HA3 – some sol's

1.9.5 (a) true, since A non-singular

(b) false $\det\left(2\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix}\right) = 4$

(c) false $\det\left(\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix}\right) = 4 \neq 1+1$

(d) true $A^{-T} = (A^{-1})^T \ldots$

(e) true (1.53)

(f) false $A = \begin{pmatrix}0&1\\0&0\end{pmatrix}, B = \begin{pmatrix}0&0\\1&0\end{pmatrix} \Rightarrow -1 \neq 0$

(g) true : n pivots in row-echelon form implies $\det A \neq 0$ and rank $A = n$

(h) true $AB = 0 \Rightarrow B = A^{-1}\cdot 0 = 0$

2.1.2 Check axioms : for exaple

$$c\,'\cdot'\left((x_1,y_1)\,'+'\,(x_2,y_2)\right) \underset{def}{=} c\cdot\left((x_1x_2,\, y_1y_2)\right)$$

$$\underset{def}{=} \left((x_1x_2)^c, (y_1y_2)^c\right) \underset{law}{\overset{exp'}{=}} \left(x_1^c x_2^c,\, y_1^c y_2^c\right)$$

$$\underset{def}{=} \left(x_1^c, y_1^c\right)\,'+'\,\left(x_2^c, y_2^c\right) \underset{def}{=} c\cdot(x_1,y_1) + c\cdot(x_2,y_2)$$

....

$\boxed{2.1.10}$ $\underline{a} + \underline{b} = (a_1+b_1, a_2+b_2, a_3+b_3, \ldots\ldots)$

$c \cdot \underline{a} = (ca_1, ca_2, \ldots\ldots)$

axioms hold since they hold component-wise.

$\boxed{2.2.2}$

(a) $0 \notin V$

(b) yes, sol' $x+y=0$

(c) yes.

(d) yes

(e) $0 \notin V$

(f) $-\underline{x} \notin V$

(g) yes

(h) $\begin{pmatrix}1\\1\\1\end{pmatrix} \in V$, $\begin{pmatrix}2\\2\\2\end{pmatrix} \notin V$ $\Rightarrow$ no

(i) yes, $V = \{0\}$

(j) $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \in V$, $\begin{pmatrix}1\\1\\0\end{pmatrix} \notin V \Rightarrow$ no

$\boxed{2.2.8}$ We always have $0 \in V$

$\Rightarrow A \cdot 0 = b \Rightarrow b = 0$

$\boxed{2.2.12}$

(a) no $\begin{pmatrix}1&0\\0&1\end{pmatrix} + \begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ is not regular

(b) same example, no

(c) no $\begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}0&0\\0&1\end{pmatrix}$ is nonsingular

(d) yes

(e) no $(0 \in V)$

f) - h) yes

$\boxed{12.2.15}$ only (e) does not, $0 \in V$