HA 5

3.1.4 (b) symmetry

$$\langle v,w\rangle = 4v_1w_1 + 2v_1w_2 + 2v_2w_1 + 4v_2w_2 + v_3w_3$$

$$= 4w_1v_1 + 2w_1v_2 + 2w_2v_1 + 4w_2v_2 + w_3v_3$$

$$= \langle w,v\rangle$$

bilinear $\langle cu+dv, w\rangle = 4(cu_1+dv_1)\cdot w_1 + 2(cu_1+dv_1)w_2$

$$+ 2(cu_2+dv_2)w_1 + 4(cu_2+dv_2)w_2$$

$$+ (cu_3+dv_3)w_3$$

$$= c\cdot(4u_1w_1 + 2u_1w_2 + 2u_2w_1 + 4u_2w_2 + u_3w_3)$$

$$+ d\,(4v_1w_1 + 2v_1w_2 + 2v_2w_1 + 4v_2w_2 + v_3w_3)$$

$$= c\langle u,w\rangle + d\langle v,w\rangle$$

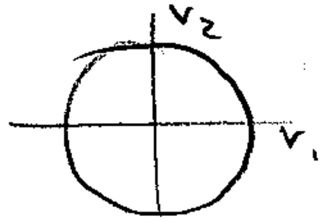
$\langle u, cv+dw\rangle$ similar...

positive: $\langle v,v\rangle = 4v_1^2 + 4v_1v_2 + 4v_2^2 + v_3^2$

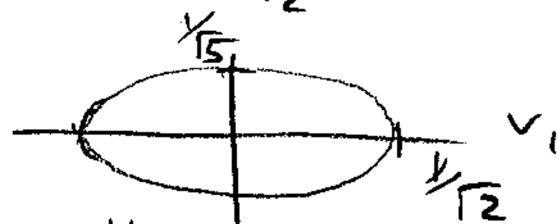
$$= (2v_1+v_2)^2 + 3v_2^2 + v_3^2$$

$$= 0 \text{ only if } v_1 = v_2 = v_3 = 0$$

3.1.5 (a) $v_1^2+v_2^2=1$

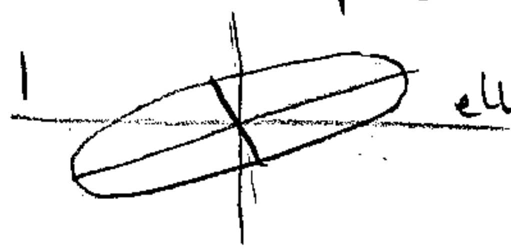


(b) $2v_1^2+5v_2^2=1$



ellipse

(c) $v_1^2-2v_1v_2+4v_2^2=1$

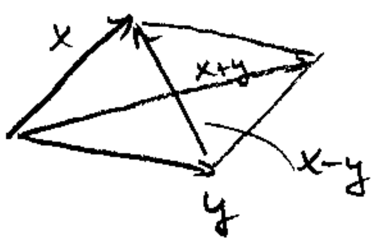


ellipse

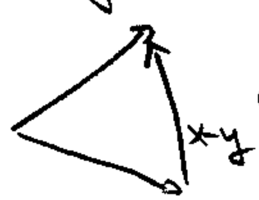
(d) (b) is the stadard form of a ellipse

3.1.12. $(x+y,x+y)+(x-y,x-y) = (x,x)+2(x,y)+(y,y)$
$+(x,x)-2(x,y)+(y,y)$

$=2(x,x)+2(y,y)$



Pythagoras for



and

with correction $2(x,y)$
for not rectangular
triangles

3.1.20

(c) $(f,g) = \int_0^1 x \cdot (1+x^2) \cdot x \, dx$

$= \frac{1}{3} + \frac{1}{5}$

3.1.21

(a) $w = e^{-x}$, positive ✓

(b) $w(x) = x$ not positive, $f = g = x$

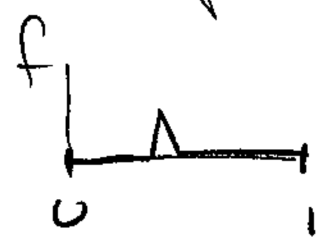
$\Rightarrow \langle f, f \rangle = \int_{-1}^{1} x^3 = 0$ (not positive)

(c) $w(x) = x+2$ positive on $[-1,1]$

(d) $w(x)$ not strictly positive, but still

$\int f^2(x) \cdot x^2 = 0$ implies $f(x) = 0$ for all x, so yes

3.1.23 (a) no, not positive



$\langle f, f \rangle = 0$

(b) yes, positive, symmetric, and bilinear

3.1.26 (a) no, $f(x)=1$, $\langle f,f\rangle = 0$

(b) yes: $\langle f,f\rangle = 0$

implies $f'(x)=0$ for all x.

Since $f(0)=0$, this gives $f=0$ for all x.

3.2.2 (b) $\|(1,1,1,1)^T\| = \sqrt{4} = 2$

$\|(a_1,a_2,a_3,a_4)^T\| = 2$ also

$$\cos\theta = \frac{\langle (1,1,1,1)^T, (a_1,\dots,a_4)^T\rangle}{4} = \begin{cases} -1 & a_j = -1 \\ -1/2 & \\ 0 & \\ 1/2 & \\ 1 & a_j = +1 \end{cases}$$

$$\theta = \begin{cases} \pi \\ \pi/3 \\ \pi/2 \\ -\pi/3 \\ 0 \end{cases}$$

3.2.3 Verify that all differences

$(0,0,0)-(1,1,0)$ $\quad$ $(1,1,0)-(1,0,1)$

$(0,0,0)-(1,0,1)$ $\quad$ $(0,1,1)-(1,0,1)$

$(0,0,0)-(0,1,1)$ $\quad$ $(1,1,0)-(0,1,1)$

have length $\sqrt{2}$ ✓

Common angle: $\cos\theta = \frac{1}{2} = \pi/3$

Center angle:

$v_1 = \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$, $v_2 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$, $\|v_j\| = \frac{\sqrt{3}}{2}$

$$\cos\theta = \frac{-1/4}{3/4} = -\frac{1}{3} \sim \theta \sim 1.91\ldots$$

3.2.13. $\|1\| = \sqrt{\int_0^\pi 1\,dx} = \sqrt{\pi}$

$$\|\cos x\| = \sqrt{\int_0^\pi \cos^2 x\,dx} = \sqrt{\pi/2}$$

$$\langle 1, \cos x\rangle = \int_0^\pi \cos x = 0 \leadsto \theta = \pi/2$$

$$\|\sin x\| = \sqrt{\pi/2}, \quad \langle 1, \sin x\rangle = \int_0^\pi \sin x = 2$$

$$\Rightarrow \cos\theta = \frac{2}{\pi/\sqrt{2}} = \frac{2\sqrt{2}}{\pi} \leadsto 0.45$$

$$\langle \sin x, \cos x\rangle = \int_0^\pi \sin x \cos x = \frac{1}{2}\int_0^\pi \sin 2x = 0$$

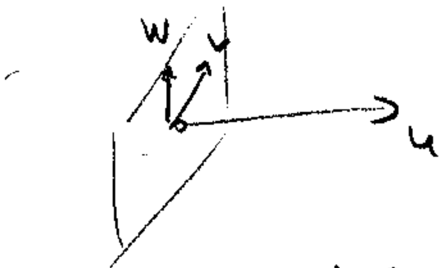
$$\leadsto \theta = \pi/2$$

3.2.19 $w \in W$ if $w_1 + 2w_2 - w_3 + 3w_4 = 0$

↳ linear eq', find basis for kernel!

$$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3.2.20



$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ w = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

↳ lin. indep

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ w = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

↳ lin. dep!

3.2.27 $\int_{-1}^{1} (ax^2 + bx + c) = 0, \ \int_{-1}^{1} ax^3 + bx^2 + cx = 0$

$\Rightarrow \ a/3 + c = 0, \ b = 0 \ \hookrightarrow \ a = 3, \ c = -1$

$$\boxed{p(x) = 3x^2 - 1}$$