HA-6 — some solutions

3.4.22

(v) (a) $\begin{pmatrix} 9 & 6 & 3 \\ 6 & 6 & 0 \\ 3 & 0 & 3 \end{pmatrix}$

(b) Gauss-elimination

$$\begin{matrix} 9 & 6 & 3 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{matrix} \rightarrow \begin{matrix} 9 & 6 & 3 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{matrix}$$ — not regular

$\Rightarrow$ semi-definite only

(c) null vector: only one $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

3.4.25

$$\langle 1,1\rangle = \int_0^1 1 = 1$$

$$\langle 1,e^x\rangle = \int_0^1 e^x = e-1$$

$$\langle 1,e^{2x}\rangle = \int_0^1 e^{2x} = \tfrac{1}{2}(e^2-1)$$

$$\langle e^x,e^x\rangle = \int e^{2x} = \tfrac{1}{2}(e^2-1)$$

$$\langle e^x,e^{2x}\rangle = \int e^{3x} = \tfrac{1}{3}(e^3-1)$$

$$\langle e^{2x},e^{2x}\rangle = \int e^{4x} = \tfrac{1}{4}(e^4-1)$$

$$K = \begin{pmatrix} 1 & e-1 & \frac{1}{2}(e^2-1) \\ e-1 & \frac{1}{2}(e^2-1) & \frac{1}{3}(e^3-1) \\ \frac{1}{2}(e^2-1) & \frac{1}{3}(e^3-1) & \frac{1}{4}(e^4-1) \end{pmatrix}$$

$K>0$ yes, since fcts are linearly independent

$$c_1 1 + c_2 e^x + c_3 e^{2x} = 0 \quad \text{on } [0,1]$$

$$\Rightarrow \quad c_1 = c_2 = c_3 = 0$$

3.4.27

$$\langle 1,1\rangle = \int_{-1}^{1} 1 = 2$$
$$\langle 1,x\rangle = \int x = 0$$
$$\langle 1,x^2\rangle = \int x^2 = 2/3$$
$$\langle 1,x^3\rangle = \int x^3 = 0$$

$$\langle x,x\rangle = \int x^2 = 2/3$$
$$\langle x,x^2\rangle = \int x^3 = 0$$
$$\langle x,x^3\rangle = \int x^4 = 2/5$$
$$\langle x^2,x^3\rangle = \int x^5 = 0$$
$$\langle x^3,x^3\rangle = 2/7$$

$$K = 2\cdot\begin{pmatrix} 1 & 0 & 1/3 & 0 \\ 0 & 1/3 & 0 & 1/5 \\ 1/3 & 0 & 1/5 & 0 \\ 0 & 1/5 & 0 & 1/7 \end{pmatrix}$$

again, $K>0$
since linearly indep'
(book shows lin. indep' over $x\in[0,1] \Rightarrow$ lin. indep' on $[-1,1]$)

3.5.1 (e)

$$\begin{matrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{matrix} \rightarrow \begin{matrix} 2 & 1 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{matrix} \rightarrow \begin{matrix} 2 & 1 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{4}{3} \end{matrix} \rightarrow \begin{matrix} 2 & 1 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{5}{4} \end{matrix}$$

pivots > 0, regular $\rightarrow K > 0$

3.5.3.

$$\begin{matrix} 1 & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 0 \\ 0 & c-1 & 1 \\ 0 & 1 & 1 \end{matrix}$$

$c > 1$

$$\hookrightarrow \begin{matrix} 1 & 1 & 0 \\ 0 & c-1 & 1 \\ 0 & 0 & \cancel{0}\, 1-\frac{1}{c-1} \end{matrix} \rightarrow 1 - \frac{1}{c-1} > 0$$

$$\hookrightarrow c-1-1 > 0$$

$$\boxed{c > 2}$$

3.5.7 (d)

$$3x_1^2 - x_2^2 + 5x_3^2 + 4x_1x_2 - 7x_1x_3 + 9x_2x_3$$

$$\hookrightarrow K = \begin{pmatrix} 3 & 2 & -\frac{7}{2} \\ 2 & -1 & \frac{9}{2} \\ -\frac{7}{2} & \frac{9}{2} & 5 \end{pmatrix} \hookrightarrow \begin{matrix} 3 & 2 & -\frac{7}{2} \\ 0 & -\frac{7}{3} & \ldots \\ & & \end{matrix}$$

not > 0

3.5.19

(c) $\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 14 \end{matrix} \rightsquigarrow \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & -3 & 13 \end{matrix} \rightsquigarrow \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 4 \end{matrix}$

$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \rightsquigarrow S = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$

$K = (LS)(LS)^T, \quad LS = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$

4.2.4

(a) $\begin{pmatrix} 1 & b \\ 0 & 4-b^2 \end{pmatrix} \rightsquigarrow |b| < 2$

(b) $L = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \ D = \begin{pmatrix} 1 & 0 \\ 0 & 4-b^2 \end{pmatrix}, \ S$

(c) $p(x,y) = x^T A x - 2 x^T \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|b| > 2$: minimum $\to -\infty$

$|b| < 2$: minimum from

$$A x_* = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightsquigarrow x_* = \frac{1}{4-b^2} \begin{pmatrix} -b \\ 1 \end{pmatrix}$$

$|b| = 2$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \notin \text{rng}$! $\rightsquigarrow$ minimum $\to -\infty$

4.2.5.

(c) $3x_1^2 - 2x_1x_2 + 2x_1x_3 + 2x_2^2 - 2x_2x_3 + 3x_3^2$

$\cancel{+}2(x_1 - 2x_3) - 3$

$$\begin{matrix} 3 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 3 \end{matrix} \rightsquigarrow \begin{matrix} 3 & -1 & 1 \\ 0 & 5/3 & -2/3 \\ 0 & -2/3 & 5/3 \end{matrix} \rightsquigarrow \begin{matrix} 3 & -1 & 1 \\ 0 & 5/3 & -2/3 \\ 0 & 0 & 7/5 \end{matrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 2/5 & 1 \end{pmatrix} \rightsquigarrow \text{ex. global min.}$$

$K x^* = f \rightsquigarrow$

$$x = \left(7/12, -2/12, -11/12\right)^T$$