HA 8 - solutions

<u>5.4.11</u> a) $q_0 = 1, \quad \|q_0\|^2 = 1$ | $\langle t^j, t^k\rangle = \int_1^2 t^{k+j} = \frac{2^{k+j+1}-1}{k+j+1}$

$$q_1 = t - \frac{\langle t, 1\rangle}{\|q_0\|^2}\cdot q_0 = t - 3/2$$

$q_2 =$

$$\|q_1\|^2 = \int_1^2 (t-\tfrac{3}{2})^2 = \frac{2(\frac{1}{2})^3}{3} = \frac{1}{12}$$

$$q_2 = t^2 - \frac{\langle t^2, q_1\rangle}{\|q_1\|^2}\cdot q_1 - \frac{\langle t^2, q_0\rangle}{\|q_0\|^2} q_0$$

$$= t^2 - \frac{1/4}{1/12}\cdot(t - 3/2) - \frac{7/3}{1}\cdot 1 = t^2 - 3t + 13/6$$

$$\|q_2\|^2 = 1/180$$

$$q_3 = t^3 - \frac{\overbrace{\langle t^3, q_2\rangle}^{=1/40}}{\underbrace{\|q_2\|^2}_{=1/180}} q_2 - \frac{\overbrace{\langle t^3, q_1\rangle}^{=23/40}}{\underbrace{\|q_1\|^2}_{=1/12}} q_1 - \frac{\langle t^3, q_0\rangle}{\|q_0\|^2} q_0$$

$$= t^3 - 9/2\left(t^2 - 3t + 13/6\right) - \frac{69}{10}\left(t - \frac{3}{2}\right) - \frac{15}{4}$$

5.4.20

a) $$\int_{-1}^{1} \frac{\cos(n \arccos t)\ \cos(m \arccos t)}{\sqrt{1-t^2}}\, dt$$

$$= \int_0^{\pi} \cos(nx)\cos(mx)\,dx = 0$$

for $m \neq n$

b) $$\|T_n\|^2 = \int_0^{\pi} \cos^2(nx)\,dx = \pi/2$$

5.5.1 c) $x-y-z=0 \iff \left\langle \begin{pmatrix}1\\-1\\-1\end{pmatrix}, \begin{pmatrix}x\\y\\z\end{pmatrix} \right\rangle = 0$

$\iff$ only v_2 is orthogonal

e) orthogonal to range $\Rightarrow$ orthogonal to both $\begin{pmatrix}-3\\3\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}$: v_1 ✓ $\cancel{v_2}$

$\cancel{v_3}$ $\cancel{v_4}$

5.5.3

$$c_1 = \frac{\left\langle \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix} \right\rangle}{\left\| \begin{pmatrix}3\\2\\1\end{pmatrix} \right\|^2} = \frac{10}{14}$$

(Thm 5.37)

$\begin{pmatrix}3\\2\\1\end{pmatrix} \perp \begin{pmatrix}2\\-2\\-2\end{pmatrix}$

$$c_2 = \frac{\left\langle \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\-2\\-2\end{pmatrix} \right\rangle}{\left\| \begin{pmatrix}2\\-2\\-2\end{pmatrix} \right\|^2} = -\frac{8}{12} = -\frac{2}{3}$$

$$u = \frac{10}{14}\begin{pmatrix}3\\2\\1\end{pmatrix} - \frac{2}{3}\begin{pmatrix}2\\-2\\-2\end{pmatrix}$$

5.5.28

$P_0 = 1 \qquad P_2 = \frac{3}{2}t^2 - \frac{1}{2}$

$P_1 = t \qquad P_3 = \frac{5}{2}t^3 - \frac{3}{2}t$

$$c_0 = \frac{\langle t^4, P_0 \rangle}{\|P_0\|^2} = \frac{2/5}{2} = 1/5$$

$$c_1 = \frac{\langle t^4, P_1 \rangle}{\|P_1\|^2} = 0$$

$$c_2 = \frac{\langle t^4, P_2 \rangle}{\|P_2\|^2} = \frac{8/35}{2/5} = 4/7$$

$$c_3 = \text{— —} = 0$$

(a), (b) sae aswer

$c_0 P_0 + c_2 P_2$

$= \frac{1}{5} + \frac{4}{7}\left(\frac{3}{2}t^2 - \frac{1}{2}\right)$

5.6.2 (a) $\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$

(b) $-2x + y + 3z = 0 \Rightarrow$

$$\begin{pmatrix} 3/2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix}$$

(c) Row Echelon gives $\dim Rg = \cancel{\varepsilon}\ 2$

spanned by $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$, for ex.

$$\left.\begin{aligned} x - 2y - z &= 0 \\ 2x \quad + 2z &= 0 \end{aligned}\right\} \quad \underline{\underline{\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}}$$

(d) $\dim \operatorname{coker} = 1$, $\operatorname{coker} = (Rg)^{\perp}$

$\hookrightarrow \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ basis

5.6.4.

(a) $w = \langle v, \begin{pmatrix} -3 \\ 1 \end{pmatrix} \rangle / \|\begin{pmatrix} 3 \\ 1 \end{pmatrix}\|^2 \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$= -\frac{1}{10} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$z = v - w = \frac{1}{10} \begin{pmatrix} 7 \\ 21 \end{pmatrix}$

(d) range spanned by column vectors, linearly independent $\Rightarrow$ range $= \mathbb{R}^3$

$\Rightarrow w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad z = 0$

(e) basis for kernel : row echelon

$\begin{matrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 1 \end{matrix} \rightsquigarrow \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \xrightarrow{\text{orthogonalize}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -2 \\ 1/2 \\ -1/2 \\ 1 \end{pmatrix}$

$w = \langle v, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \rangle / \|\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}\|^2 \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \langle v, \begin{pmatrix} -2 \\ 1/2 \\ -1/2 \\ 1 \end{pmatrix} \rangle / \|..\|^2 \cdot \begin{pmatrix} -2 \\ 1/2 \\ -1/2 \\ 1 \end{pmatrix}$

$= -\frac{1}{11/2} \begin{pmatrix} -2 \\ 1/2 \\ -1/2 \\ 1 \end{pmatrix}, \quad z = v - w$

5.6.11 $w \in W_2^\perp \Rightarrow \langle w, w_2 \rangle = 0 \quad \forall w_2 \in W_2$

$\Rightarrow \langle w, w_1 \rangle = 0 \quad \forall w_1 \in W_1 \subset W_2$

$\Rightarrow w \in W_1^\perp$, so $W_1^\perp \supset W_2^\perp$

5.6.22

(a) $A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$ $b = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

$\ker A^T = \ker \begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\langle b, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rangle = 0 \Rightarrow$ compatible, ex. solution

(d) $\begin{pmatrix} 1 & 3 & 5 \\ -1 & 4 & 9 \\ 2 & 3 & 4 \end{pmatrix} = A$, $A^T = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 4 & 3 \\ 5 & 9 & 4 \end{pmatrix}$

$\leadsto \begin{matrix} 1 & -1 & 2 \\ 0 & 7 & -3 \\ 0 & 14 & -6 \end{matrix} \leadsto \begin{matrix} 1 & -1 & 2 \\ 0 & 7 & -3 \\ 0 & 0 & 0 \end{matrix} \leadsto$ kernel $\begin{pmatrix} -11/7 \\ 3/7 \\ 1 \end{pmatrix}$

$b = (3, 11, 0)^T \Rightarrow$ compatible ✓