HA 9

7.1.1 (a) yes, (b) no (c) no (d) yes (e) no (f) no (g) no

e.g. (e) $F(2,2,2) = 8$

$2 \cdot F(1,1,1) = 2 \neq F(2,2,2)$

(g) $F(0,0,0) = 1 \neq 0$

7.1.2 (a) yes (b) no (c) no (d) yes (e) no (f) yes

e.g. (d) $F\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \end{pmatrix} = \begin{pmatrix} 3(y_1+y_2) \\ 2(x_1+x_2) \end{pmatrix} = \begin{pmatrix} 3y_1 \\ 2x_1 \end{pmatrix} + \begin{pmatrix} 3y_2 \\ 2x_2 \end{pmatrix} =$

$= F\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + F\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$F\begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix} = \begin{pmatrix} \alpha 3y \\ \alpha 2x \end{pmatrix} = \alpha \begin{pmatrix} 3y \\ 2x \end{pmatrix} = \alpha F\begin{pmatrix} x \\ y \end{pmatrix}$

(c) $F\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \neq 2 \cdot F\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

7.1.3 (a) $F(0) \neq 0$

(b) $F\begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix} = \alpha^2 F\begin{pmatrix} x \\ y \end{pmatrix}$, not $\alpha F\begin{pmatrix} x \\ y \end{pmatrix}$

(c) $F\begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix} = |\alpha| F\begin{pmatrix} x \\ y \end{pmatrix}$, not $\alpha F\begin{pmatrix} x \\ y \end{pmatrix}$,

e.g. $\alpha = -1$

(d) $F\begin{pmatrix} \pi/2 \\ 0 \end{pmatrix} + F\begin{pmatrix} \pi/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \pi/2 \end{pmatrix} + \begin{pmatrix} 1 \\ \pi/2 \end{pmatrix} \neq F\begin{pmatrix} \pi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \pi \end{pmatrix}$

(e) $F\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

7.1.6 $v = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$v_1 = c_1 + c_2 \quad \to \quad c_1 = \frac{v_1 + v_2}{2}$

$v_2 = c_1 - c_2 \qquad\quad c_2 = \frac{v_1 - v_2}{2}$

$L(v) = L\left(c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) =$

$= c_1 L\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 L\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= \frac{v_1 + v_2}{2} \cdot 2 + \frac{v_1 - v_2}{2} \cdot 3$

$= \frac{5}{2} v_1 - \frac{1}{2} v_2$

3

7.1.11

No, $N(-v) = N(v) \neq -N(v)$
in general

7.1.12

No $Q(2v) = (2v)^T K(2v) = 4v^T K v$

$= 4Q(v) \neq 2Q(v)$, when $Q(v) \neq 0$

7.1.15

(a) yes (d) no (g) yes

(b) no (e) no (h) yes

(c) yes (f) yes (i) yes

e.g.

(b) $L[O] = I \neq O$

(c) $(A+B)^T = A^T + B^T$, $(\alpha A)^T = \alpha A^T$

(d) $L2[O]$ not even defined

(e) $\det(2 \cdot A) \neq 2 \cdot \det(A)$
$= \det(A) \cdot 2^n$

(h) $(A+B)v = Av + Bv$

$(\alpha A)v = \alpha(Av)$

7.1.19 (a) yes (b) no (c) yes (d) yes (e) yes (f) yes (g) no (h) yes (i) yes (j) yes (k) no (l) yes (m) no (n) yes (o) "no" (p) no (q) yes (r) yes (s) no

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(b) $L(2) = 4 \neq 2 \cdot L(1) = 2$

(f) $L[f+g] = (f+g)(x+2) = f(x+2) + g(x+2)$
$= L[f] + L[g]$

$L[\alpha f] = \alpha \cdot f(x) = \alpha L[f]$

(k) $L[2] = 2 \cdot \log 2 \neq 2L[1] = 0$

(o) not defined for $f(y) = 1$, $x \in \mathbb{R}$
ok with target space $C^1([1,2])$, for instance.

(5)

(p) $L[2] = \int_0^2 y\,dy = 2 \neq$

$2 \cdot L[1] = 2\int_0^1 y\,dy = 1$

(t) $L[f+g] = \int_{-1}^{1} [f(y)+g(y) - (f(0)+g(0))]\,dy$

$= \int_{-1}^{1} [f(y) - f(0)]\,dy + \int_{-1}^{1} [g(y) - g(0)]\,dy$

$= L[f] + L[g]$

(s) $L[\alpha f] = \alpha L[f]$ ...

(s) $L[0] = \int_{-1}^{x} (-y)\,dy = -x^2/2 + \frac{1}{2} \neq 0$

7.1.28 L_1, L_2 in subset so

$L_1\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}, \quad L_2\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$\Rightarrow (L_1+L_2)\begin{pmatrix}1\\0\end{pmatrix} = L_1\begin{pmatrix}1\\0\end{pmatrix} + L_2\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$(\alpha L_1)\begin{pmatrix}1\\0\end{pmatrix} = \alpha\left(L_1\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}0\\0\end{pmatrix}$

$\Rightarrow$ yes, Subspace

7.1.29 no: $L_{\gamma}\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$$\Rightarrow (2L)\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\left(L\begin{pmatrix}0\\1\\0\end{pmatrix}\right) = 2\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\2\\0\end{pmatrix}$$

So $2L$ not in set when L is, not a subspace.

7.1.38

(a) $L\begin{pmatrix}v_1\\v_2\end{pmatrix} = L(v_1e_1 + v_2e_2) =$

$$= v_1\begin{pmatrix}1\\-3\end{pmatrix} + v_2\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}v_1 - v_2\\-3v_1 + 2v_2\end{pmatrix}$$

$$= \begin{pmatrix}1 & -1\\-3 & 2\end{pmatrix}\begin{pmatrix}v_1\\v_2\end{pmatrix}$$

(b) $M\begin{pmatrix}v_1\\v_2\end{pmatrix} = \begin{pmatrix}-1 & 0\\-3 & 2\end{pmatrix}\begin{pmatrix}v_1\\v_2\end{pmatrix}$

(c) $\begin{pmatrix}v_1\\v_2\end{pmatrix} = c_1\begin{pmatrix}1\\-3\end{pmatrix} + c_2\begin{pmatrix}-1\\2\end{pmatrix}$

$$\begin{matrix} 1 & -1 & v_1 \\ -3 & 2 & v_2 \end{matrix} \quad \hookrightarrow \quad \begin{matrix} 1 & -1 & v_1 \\ 0 & -1 & v_2+3v_1 \end{matrix}$$

$$c_2 = -v_2 - 3v_1$$

$$c_1 = -v_2 - 2v_1$$

$$N\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = N\left(c_1 \begin{pmatrix} 1 \\ -3 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right)$$

$$= c_1 N\begin{pmatrix} 1 \\ -3 \end{pmatrix} + c_2 N\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$= (-v_2 - 2v_1)\begin{pmatrix} -1 \\ -3 \end{pmatrix} + (-v_2 - 3v_1)\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2v_1 + v_2 \\ 0v_1 + v_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

(c) $$\begin{pmatrix} -1 & 0 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$$

(d) Also, $N \circ L(e_1) = N\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

$$= M(e_1)$$

$$N \circ L(e_2) = N\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} = M(e_2)$$

So coincide for $e_1, e_2 \Longrightarrow$ same linear maps

7.1.51

(a) yes, halven the length of the vector

(b) yes, ~~count~~ clockwise rotation by $45°$

(c) yes, reflection through the y-axis

(d) no, $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}0\\0\end{pmatrix}$ are mapped to $\begin{pmatrix}0\\0\end{pmatrix}$ hence not 1-1

(e) yes, shear back $\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$

7.1.58

$A = \begin{pmatrix}1\\1\end{pmatrix}$ has left-inverse

$L = (1 \quad 0)$, or $L = (0 \quad 1)$,

or $L = (\alpha \quad 1-\alpha)$, for any α.

7.2.24

(a) $\begin{pmatrix}1 & -4\\-2 & 3\end{pmatrix} = A_0$

(b) $S = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, $S^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$

$$A = S^{-1} A_0 S = \begin{pmatrix} 1 & -6 \\ -\frac{4}{3} & 3 \end{pmatrix}$$

(c) $S = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$, $A = \begin{pmatrix} -1 & 0 \\ 2 & 5 \end{pmatrix}$

(d) $S = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$, $A = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$

(e) $S = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$, $A = \begin{pmatrix} -3 & -8 \\ 2 & 7 \end{pmatrix}$

7.2.25

(a) $S = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, $A = S^{-1} A_0 S$,

$A_0 = \begin{pmatrix} -3 & 2 & 2 \\ -3 & 1 & 3 \\ -1 & 2 & 0 \end{pmatrix}$, $A = \begin{pmatrix} -3 & -1 & -2 \\ 6 & 1 & 6 \\ 1 & 1 & 0 \end{pmatrix}$

(b) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $S = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & -2 \\ 2 & -1 & 1 \end{pmatrix}$, $A = \begin{pmatrix} \frac{1}{5} & 0 & -\frac{12}{5} \\ 0 & -2 & 0 \\ -\frac{2}{5} & 0 & -\frac{1}{5} \end{pmatrix}$

7.2.26

(a) $\ker(A) = \{0\}$, $v_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $\left(v_2 = \begin{pmatrix}0\\1\end{pmatrix}\right)$ basis of complement

$w_1 = A\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix}$, $w_2 = A\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

basis of range

$T = \begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}$, $S = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$T^{-1}AS = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

(b) $\begin{pmatrix}1 & -3 & 4\\-2 & 6 & -8\end{pmatrix}$, $\ker(A) = v_3 = \begin{pmatrix}3\\1\\0\end{pmatrix}$, $v_2 = \begin{pmatrix}-4\\0\\1\end{pmatrix}$

complement $\begin{pmatrix}1\\0\\0\end{pmatrix} = v_1$

$Av_1 = \begin{pmatrix}1\\-2\end{pmatrix} = w_1$

basis of $\ker(A^T)$: $\begin{pmatrix}1 & -2\\-3 & 6\\4 & -8\end{pmatrix} \to \begin{matrix}1 & -2\\0 & 0\\0 & 0\end{matrix} \to \begin{pmatrix}0\\1\end{pmatrix} = w_2$

$T = \begin{pmatrix}1 & 0\\-2 & 1\end{pmatrix}$, $S = \begin{pmatrix}1 & -4 & 3\\0 & 0 & 1\\0 & 1 & 0\end{pmatrix}$

$T^{-1}AS = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$