

Quiz 4 - solutions

(a) no, $\langle x, x \rangle = 0$ for $x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(b) no, not bilinear $\langle x, 0 \rangle = x_2^2$
 $\langle 2x, 0 \rangle = 4x_2^2$

(c) yes

(d) no, as in (b)

(e) no, $\langle x, x \rangle = (x_1 + 1)^2 + (x_2 - 1)^2 + x_3^2 = 0$

for $x_1 = -1, x_2 = 1, x_3 = 0$

(also not bilinear)

(f) no, $\langle x, x \rangle = (x_1^2 + x_2^2 + x_3^2) \cdot \cos \pi = -(x_1^2 + x_2^2 + x_3^2)$

$$\leq 0$$

(g) no, $\langle x, y \rangle \neq \langle y, x \rangle$

(h) no, $\langle x, x \rangle = x_3^2 + (2x_2 - x_1)^2 = 0$

for $x_3 = 0, x_1 = x_2 = 1$

$$\begin{aligned}
 2) \quad \|x+y\|^2 + \|x-y\|^2 &= \langle x+y, x+y \rangle + \langle x-y, x-y \rangle \\
 &= \langle x, x \rangle + \langle y, y \rangle + 2\langle x, y \rangle + \langle x, x \rangle \\
 &\quad + \langle y, y \rangle - 2\langle x, y \rangle = \\
 &= 2\|x\|^2 + 2\|y\|^2
 \end{aligned}$$

$$\begin{aligned}
 3) \quad a) \quad \langle 1, x \rangle &= \int_0^1 x = \frac{1}{2} \\
 \|1\| &= 1, \quad \|x\| = \sqrt{\int_0^1 x^2} = \frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\cos \varphi = \frac{\frac{1}{2}}{\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

$$b) \quad \int_0^1 f(x) = 0, \quad \int_0^1 f(x) \cdot x = 0$$

$$f(x) = ax^2 + bx + c, \quad a \rightarrow \frac{a}{3} + \frac{b}{2} + c = 0$$

$$\frac{a}{4} + \frac{b}{3} + \frac{c}{2} = 0$$

$$\text{for ex.: } a=6, \quad b=-6, \quad c=1$$

$$c) \quad \text{no, } \langle 1, 1 \rangle = 0$$

d) yes, bilinear, symmetric ✓

$$\langle f, f \rangle = 0 \Rightarrow f' = 0, \quad \int f = 0 \Rightarrow f = 0$$