

Quiz 6 - solutions

$$1) \quad w_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \|w_1\|^2 = 5$$

$$w_2 = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} - \frac{5}{5} \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \|w_2\|^2 = 14$$

$$w_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - 0 \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \frac{3}{14} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \\ 5 \end{pmatrix} / 14$$

$$w_1^2 = \begin{pmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{pmatrix}, \quad w_2^2 = \begin{pmatrix} 2/\sqrt{14} \\ -1/\sqrt{14} \\ 3/\sqrt{14} \end{pmatrix}$$

$$w_3^2 = \begin{pmatrix} -6 \\ 3 \\ 5 \end{pmatrix} / \sqrt{70}$$

$$2) \quad A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$\tau_{11} = \sqrt{5}$$

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & 3 \\ \frac{2}{\sqrt{5}} & 4 \end{pmatrix}$$

$$\tau_{12} = \frac{3}{\sqrt{5}} + \frac{8}{\sqrt{5}} = \frac{11}{\sqrt{5}}$$

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & 3 - \frac{11}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & 4 - \frac{22}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{4}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}$$

$$\tau_{22} = \frac{\sqrt{20}}{\sqrt{5}}$$

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix} = Q$$

$$\begin{pmatrix} \sqrt{5} & \frac{11}{\sqrt{5}} \\ 0 & \frac{\sqrt{20}}{\sqrt{5}} \end{pmatrix} = R$$

$$3) \quad A = \begin{pmatrix} 0 & 2 \\ 1 & 1 \\ 6 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$K = A^T A = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 1 & 1 \\ 6 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 14 \end{pmatrix}$$

$$f = A^T b = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

$$Kx = f \quad \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 14 & 6 & 6 & 13 & 5 \end{array}$$

$$x = \begin{pmatrix} 8/13 \\ 5/13 \end{pmatrix} \rightarrow \text{closest pt.}$$

$$Ax = \begin{pmatrix} 10/13 \\ 13/13 \\ 15/13 \end{pmatrix}$$

4) b) is a permutation matrix * $\begin{pmatrix} 1 & & & 0 \\ & \ddots & & \\ & & 1 & \\ 0 & & & -1 \end{pmatrix}$,
hence orthogonal