Math 8501, Fall 2018

Theory of Ordinary Differential Equations

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— Exercises: Dunford's integral —

Consider a matrix $A \in L(\mathbb{C}^n, \mathbb{C}^n)$. Show the following properties of the Dunford-Taylor integral.

(1) $P = \int_\Gamma (\lambda - A)^{-1} \mathrm{d}\lambda$ is a projection, that is, $P^2 = P$, for any simple closed curve Γ, oriented counter-clockwise, such that $A - \lambda$ is invertible for $\lambda \in \Gamma$.

(2) $PA = AP$ and $\mathrm{spec}\,(A|_{\mathrm{Rg}\,(P)}) = \mathrm{spec}\,A \cap \mathrm{int}(\Gamma)$.

(3) $P = \mathrm{id}$ when $\Gamma = \{|\lambda| = R\}$, $R > \|A\|$.

(4) $\mathrm{e}^{At} P = P \mathrm{e}^{At} = \int_\Gamma \mathrm{e}^{\lambda t} (\lambda - A)^{-1} \mathrm{d}\lambda$.

(5) The spectrum of a matrix depends continuously on the matrix entries, when considered as a subset of $\mathbb{C}$ with the symmetric Hausdorff distance.

You can turn this worksheet in for extra credit. You may use information from [Kato] *or* [Lunardi] *but you need to give complete proofs yourself (assuming basic complex analysis, like the calculus of residues).*