Math 8501, Fall 2018

Theory of Ordinary Differential Equations

Arnd Scheel, VinH 509, phone 625-4065, scheel@umn.edu

— *Homework 1* —

(1) A dog has grabbed a sausage in the origin of the (x, y)-plane and starts running along the x-axis with speed 1. At the same time, a second dog starts running off at $x = 0, y = d$, with the same speed 1, always in the direction of the first dog. How close do the two dogs get?

(2) We would like to solve the pendulum equation $x'' = \sin(x) = 0$, or

$$x' = y, \qquad y' = -\sin(x),$$

with initial condition $x(0) = 0, y(0) = 3/2$, using many methods.

(i) Use Mathematica's DSolve to find the solution in terms of special functions. Plot the solution (resolution!). Find the numerical value at times $t = 1, t = 10$.

(ii) Compute (still using Mathematica) the Taylor expansion of this "explicit" solution to order 7 and compare with the numerical value from (i). Comment!

(iii) Use the Hamiltonian function from class to reduce our equation to a first-order equation. Try to solve this first-order equation using Mathematica using symbolic integration. Solve the first-order equation using DSolve. Then compute the value of the solution at time $t = 1$ and compare.

(iv) Find the Taylor expansion of the solution to this first order equation up to order 7 using the differential equation, that is, substitute the power series into the differential equation and solve recursively.

(v) Solve the initial-value problem using an explicit Euler method with step size $h = h = 10^{-j}$, $j = 1, 2, \ldots$. Use a built-in Runge-Kutta method Compare with the "exact results.

Homework is due on Monday, September 17, in class