HA 3 – Sol's

1) a) $y_{new} = y + \frac{h}{2}\left(f(y) + \frac{h}{3} f'f + \frac{h^2}{18} f''ff \right.$

$\left. + f + \frac{2h}{3} f'f + \frac{4h^2}{18} f''ff + O(h^3)\right)$

compared w/

$y(h) = y + hf + \frac{h^2}{2} f'f + \frac{h^3}{6}\left(f''f^2 + f'^2 f\right) + O(h^4)$

↳ agree at $O(h^2)$ but not $O(h^3)$

↳ $O(2)$ method.

b) $y_1 = 1 + \frac{h}{2}\left(a\left(1 + \frac{h}{3}a\right) + a\left(1 + \frac{2h}{3}a\right)\right)$

$= 1 + ha + \frac{h^2}{2}a^2$

c) $y_k = \left(1 + ha + \frac{h^2}{2}a^2\right)^k$

d) need $\left|1 + ha + \frac{(ha)^2}{2}\right| < 1$

$\tau = h a$

$\left| 1 + \tau + \frac{\tau^2}{2} \right| < 1$

$\Rightarrow \tau \in (-2, 0)$

$\Rightarrow h \in (0, -2/a)$

2) a)