Math 8501, Fall 2018

# Theory of Ordinary Differential Equations

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— *Homework 3* —

(1) Someone suggests the following method to solve $y' = f(y)$, $y(0) = y_0$:

$$y_{k+1} = y_k + \frac{\Delta t}{2} * \left( f(y_k + \frac{\Delta t}{3} f(y_k)) + f(y_k + \frac{2\Delta t}{3} f(y_k)) \right).$$

(a) Determine the order of this method: first or second order?

(b) Suppose $f(y) = ay$, $y(0) = 1$, compute $y_1$.

(c) Compute $y_k$ in this specific case explicitly.

(d) For which values of $a < 0$ does the sequence $y_k$ converge to zero.

(2) We consider the periodically forced nonlinear oscillator

$$x' = y, \qquad y' = x - x^3 + \varepsilon \cos(t).$$

(a) Draw the phase portrait for $\varepsilon = 0$ both plotting the level sets, and integrating the equation using for instance Matlab's ode45.

(b) Still setting $\varepsilon = 0$, draw a (polygonal) region enclosed by a set of initial conditions $(x_j, y_j)$ approximating a ball of radius $1/4$ around $x = 1/2, y = 0$. Integrate those initial conditions in time and observe how the shape of the ball evolves. Repeat for a ball centered at $x = 0, y = 0$. Explain why a region completely contained inside (or outside, respectively) one of the separatrices needs to stay inside of the separatrix.

(c) Repeat the previous experiments with $\varepsilon = 1$ and describe the observation. Also, draw simultaneously the time evolution of regions inside and outside of the previously shown separatrices and describe how they "mix".

*Homework is due on Monday, October 1, in class*