— HA 2 — Sol's —

1) (i) $A' = iA + A\bar{A}^2 + iA^3$

(ii) $A = A_1 + \alpha_1 A_1 \bar{A}_1^2 + \alpha_2 A_1^3$

at cubic order

$$-i\alpha_1 A_1 \bar{A}_1^2 = i\alpha_1 A_1 \bar{A}_1^2 + A_1\bar{A}_1^2$$

$$3i\alpha_2 A_1^3 = i\alpha_2 A_1^3 + iA_1^3$$

$\hookrightarrow \alpha_1 = i/2, \quad \alpha_2 = +1/2$

at quintic order

$$A_1' = iA_1 + \Big[\left(A_1 + \alpha_1 A_1 \bar{A}_1^2 + \alpha_2 A_1^3\right)\left(\bar{A}_1 + \bar{\alpha}_1 \bar{A}_1 A_1^2 + \bar{\alpha}_2 \bar{A}_1^3\right)^2 + i\left(A_1 + \alpha_1 A_1 \bar{A}_1^2 + \alpha_2 A_1^3\right)^3\Big]_{O(5)\text{-terms}}$$

$$\hookrightarrow \bar{A}_1^2 \alpha_2 A_1^3 + 3i\alpha_1 \bar{A}_1^2 A_1^3 + 2\bar{\alpha}_1 A_1 \bar{A}_1 \bar{A}_1 A_1^2 + \text{"non-resonant"}$$

$\hookrightarrow A_1' = iA_1 - (1+i) A|A|^4 \quad$ at $\mu = 0$

(iii) $\mu^2 + \mu r^2 - r^4 = 0, \quad \mu = \varepsilon^2, \; r = r_1 \varepsilon$

$\hookrightarrow -1 + r_1^2 + r_1^4 = 0 \hookrightarrow r_1 = \sqrt{\dfrac{1+\sqrt{5}}{2}}$

$$r = \sqrt{\frac{1+\sqrt{5}}{2}}\sqrt{\mu} + O(\mu), \quad \mu > 0$$

$\mu = -\varepsilon^2$: $-1 + r_1^2 + r_1^4 = 0$

$$r_1 = \sqrt{-\frac{1}{2} \pm \sqrt{\frac{1}{4}+1}} = \sqrt{\frac{\sqrt{5}-1}{2}}$$

$$r = \sqrt{\frac{\sqrt{5}-1}{2}}\sqrt{-\mu} + O(\mu) \quad , \mu < 0$$

$$\omega = 1 + \mu r^2 = 1 + \mu^2 r_1$$

2) → see paper ref's in script

3) $z = h_{20} A^2 + h_{02} \bar{A}^2 + O(3)$

$$2iA^2 h_{20} - 2i h_{02} \bar{A}^2 = -2h_{20} A^2 - 2h_{02}\bar{A}^2 + A^2 + \bar{A}^2$$

$$\rightarrow h_{20} = \frac{1}{2(1+i)} \ , \ h_{02} = \frac{1}{2(1-i)}$$

$$\rightarrow A' = iA - 3\frac{i}{2(1+i)} A|A|^2 - 3\frac{1}{2(1-i)}\bar{A}^3$$

eliminate by NF

$$r' = - \frac{3}{4} r^3 + O(4)$$

⇒ stable