MATH 1272: CALCULUS II
MIDTERM TEST II: ANSWERS TO THE SAMPLE PROBLEM SET

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Problem 1. $\pi/4$

Problem 2.

$$\frac{\pi}{6}(5\sqrt{5}-1)$$

Problem 3.

$$(\frac{3}{4}, \frac{9}{10})$$

Problem 4.

$$y = -\frac{1}{2}\ln(e^{-2} - x^2)$$

Problem 5.

$$y(t) = 200(1 - e^{-t/500})$$

Problem 6. (1) $m(100) = 200 \cdot 2^{-5/8}$
(2) $t = 160 \ln 40$

Problem 7.

$$x = -\frac{y^2}{4} + 2, \quad 0 \le y \le 2.$$

The graph is the piece of the parabola with vertex $(2, 0)$, opening to the left, enclosed between $y = 0$ and $y = 2$. The direction of increasing t should be indicated in both ways along the curve.

$$\begin{aligned} t = 0, &\quad x = 2, \quad y = 0; \\ t = \pi/2, &\quad x = 1, \quad y = 2; \\ t = \pi, &\quad x = 2, \quad y = 0. \end{aligned}$$

Problem 8.

$$y = 4(x - 5)$$

The tangent is horizontal at $t = 0$ and $t = \pm\sqrt{2}$. The second derivative is $3t^2 - 2$, while its value at $t = 2$ is 10.

Problem 9.

$$\frac{\pi^2}{2}$$

Problem 10. It is the circle of radius $3/2$ centered at $(3/2, 0)$, i.e., the one given by the equation

$$(x - \frac{3}{2})^2 + y^2 = \frac{9}{4}.$$

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The integral for the arclength is as follows:

$$\int_0^{\frac{3\pi}{2}} \sqrt{9\cos^2\theta + 9\sin^2\theta}\,d\theta = \frac{9}{2}\pi.$$

Problem 11.

$$\int_{-\cos^{-1}(1/3)}^{\cos^{-1}(1/3)} \frac{1}{2}(1 - 3\cos\theta)^2 d\theta$$

Problem 12. The vertices: $(\pm 1, 0)$, the foci: $(\pm\sqrt{2}, 0)$, the asymptotes: $y = \pm x$.

Problem 13.

$$\lim_{n\to\infty} \{\sqrt{n+2} - \sqrt{n}\} = 0$$