MATH 1272: CALCULUS II
MIDTERM TEST III: A SAMPLE PROBLEM SET

INSTRUCTOR: ALEX VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Remember that a correct answer with incorrect or no work shown is not counted.

Good luck!

Problem 1. Assume $\sum_{n=1}^{\infty} a_n$ is a series whose kth partial sum is $5 - \frac{k}{3^k}$. Find a_k. What is the sum of the series equal to?

Problem 2. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{3+2^{-n}}$$

converges.

Problem 3. Show that the following series converges and find its sum:

$$\sum_{n=1}^{\infty} \frac{2^{n-1}+3^n}{4^n}.$$

Problem 4. Determine whether the series

$$\sum_{n=1}^{\infty} 2ne^{-n}$$

converges.

Problem 5. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{5n}{2n^2-5}$$

converges.

Problem 6. Does the series

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$$

converge?

Problem 7. Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{8^n x^n}{(n+3)^2}.$$

Problem 8. Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n 3nx^n.$$

Date: November 25, 2003.

Problem 9. Evaluate the following indefinite integral as a power series and find the radius of convergence:

$$\int \frac{x}{1+x^4}\,dx.$$

Problem 10. Starting from the formula $\sum_{n=0}^{\infty}(-1)^n x^n = \frac{1}{1+x}$, valid for $|x| < 1$, find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n}.$$

Problem 11. Find the Maclaurin series for $x \cos x^3$.

Problem 12. Find an equation of the sphere that passes through $(2, -1, 7)$ and has center $(1, -3, 5)$.

Problem 13. Find the unit vector in the direction of the vector $\langle -2, 3, -1 \rangle$.

Problem 14. For which values of x are the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal?

Problem 15. Find the projection of the vector $\langle -2, 3, -1 \rangle$ onto a direction orthogonal to both of the vectors $\langle 1, 0, -2 \rangle$ and $\langle 0, 1, 1 \rangle$.