MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM TEST II WITH ANSWERS

INSTRUCTOR: SASHA VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Use Gaussian elimination to transform the augmented matrix of the following system into the echelon form. Use it to find the solutions, if there exist any.

$$
\begin{aligned}
x + y - 2z &= 0, \\
3x + 5y - 2z &= 8.
\end{aligned}
$$

Answer:

$$
\begin{bmatrix}
1 & 1 & -2 & 0 \\
0 & 1 & 2 & 4
\end{bmatrix}
$$

$$
\begin{aligned}
x &= 4t - 4 \\
y &= 4 - 2t \\
z &= t
\end{aligned}
$$

Problem 2. (1) Find the inverse of the following matrix:

$$
A = \begin{bmatrix}
0 & 2 & 1 \\
1 & 0 & 1 \\
1 & -1 & 0
\end{bmatrix}.
$$

Answer:

$$
A^{-1} = \begin{bmatrix}
1 & -1 & 2 \\
1 & -1 & 1 \\
-1 & 2 & -2
\end{bmatrix}.
$$

(2) Use the inverse of A to solve the system (another way of solving it will not be counted)

$$
\begin{aligned}
2y + z &= 1, \\
x + z &= 0, \\
x - y &= -1.
\end{aligned}
$$

Answer: $(-1, 0, 1)$.

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Problem 3. Use Cramer's rule to determine the unique solution to the system $A\mathbf{x} = \mathbf{b}$ for the following matrix and vector:

$$A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 5 \\ 2 & 3 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}.$$

   Answer: $x_1 = 1/4$, $x_2 = 1/16$, $x_3 = 21/16$.

Problem 4. Determine whether or not the set

$$S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$$

is a subspace of $\mathbb{R}^2$. Justify your answer.

   Answer: No.