

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL
EQUATIONS
SAMPLE MIDTERM TEST III**

INSTRUCTOR: SASHA VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Solve the initial value problem

$$y'' + 2y' + 2y = 4x^2, \quad y(0) = -1, \quad y'(0) = 1.$$

Answer:

$$y = e^{-x}(-3 \cos x + 2 \sin x) + 2x^2 - 4x + 2.$$

Problem 2. Determine the general solution to the following DE:

$$y'' - y = 9xe^{2x}.$$

Answer:

$$y = (3x - 4)e^{2x} + Ae^x + Be^{-x}.$$

Problem 3. An object of mass 2 kg, resting on a table next to a wall, is attached to the wall by a spring. A force of 8 N is applied to the mass, stretching the spring and moving the mass $1/2$ m from its equilibrium position. The object is then released. Suppose the resistance to the motion is numerically equal to 8 times the instantaneous velocity.

(1) Set up an IVP governing the motion of the mass.

Answer:

$$2x'' + 8x' + 16x = 0, \quad x(0) = 1/2, \quad x'(0) = 0.$$

(2) Determine the position of the mass at any time t .

Answer:

$$x = e^{-2t}/2(\cos 2t + \sin 2t).$$

(3) At what time does the mass first pass through the equilibrium position and heading away from the wall?

Answer: $t = 7\pi/8$.

Problem 4.

Determine whether the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$$

is diagonalizable. If it is, find a matrix S that diagonalizes A and determine $S^{-1}AS$.

Date: November 4, 2008.

Answer:

$$S = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}, \quad D = S^{-1}AS = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}.$$