

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL
EQUATIONS
SAMPLE MIDTERM TEST IV**

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!

Problem 1. Solve the IVP:

$$\begin{aligned}\mathbf{x}' &= \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \mathbf{x}, \\ \mathbf{x}(0) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}.\end{aligned}$$

Problem 2. Determine the general solution to the system $\mathbf{x}' = A\mathbf{x}$ for

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & -1 & 3 \\ 2 & -1 & 3 \end{bmatrix}.$$

Problem 3. Suppose we have two tanks as in the mixing problem in Figure 7.1.3 on p. 393 of the text, except that there is no inflow from or outflow to the outside, and the exchange rate between the two tanks is 2 L/min each way. Suppose that tank 1 contains 6 L of solution and tank 2 contains 12 L of solution, and that initially tank 1 contains 5g of chemical and tank 2 contains 25 g of chemical.

- (1) Determine the amount of chemical in each tank at time t .
- (2) Eventually, what will be the amount of chemical in each tank?

Problem 4. (1) Find the Laplace transform of $f(t) = \sin^2(t/2)$.

- (2) Find the inverse Laplace transform of $F(s) = \frac{1}{2s^2(s^2+1)}$. You may use the facts that $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ and $\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$ for $s > 0$.