

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL
EQUATIONS
SAMPLE MIDTERM TEST I**

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Solve the initial value problem

$$xy' + 5y = 7x^2, \quad y(2) = 5.$$

Problem 2. A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F . After 30 minutes the temperature of the cake is 140°F . When will it be 105°F ? Assume Newton's law of cooling holds.

Problem 3. A commercial fishery is estimated to have carrying capacity of 10 thousand pounds of certain kind of fish. Suppose the annual growth rate of the total fish population P , measured in thousand pounds, is governed by the logistic equation

$$\frac{dP}{dt} = \left(1 - \frac{P}{10}\right)P,$$

and initially there is a total of 2 thousand lbs of fish. What is the fish population after 1 year? (Use $10/(1 + 4e^{-1}) \approx 4.048$.)

Problem 4. (1) Use Euler's method with step size $h = 1$ to approximate the solution to the initial value problem

$$\frac{dy}{dx} = x^2 + y^3, \quad y(0) = 1,$$

on the interval $[0, 1]$.

(2) Use the improved Euler method with same step size for the same problem.