MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM TEST II

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Use Gaussian elimination to transform the augmented matrix of the following system into the echelon form. Use it to find the solutions, if there exist any.

$$\begin{aligned} x + y - 2z &= 0, \\ 3x + 5y - 2z &= 8. \end{aligned}$$

Problem 2. (1) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

(2) Use the inverse of A to solve the system (another way of solving it will not be counted)

$$\begin{aligned} 2y + z &= 1, \\ x + z &= 0, \\ x - y &= -1. \end{aligned}$$

Problem 3. Use Cramer's rule to determine the unique solution to the system $A\mathbf{x} = \mathbf{b}$ for the following matrix and vector:

$$A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 5 \\ 2 & 3 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}.$$

Problem 4. Determine whether or not the set

$$S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$$

is a subspace of $\mathbb{R}^2$. Justify your answer.

Problem 5. An object of mass 2 kg, resting on a table next to a wall, is attached to the wall by a spring. A force of 8 N is applied to the mass, stretching the spring and moving the mass 1/2 m from its equilibrium position. The object is then released. Suppose the resistance to the motion is numerically equal to 8 times the instantaneous velocity.

(1) Set up an IVP governing the motion of the mass.

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(2) Determine the position of the mass at any time t.
(3) At what time does the mass first pass through the equilibrium position and heading away from the wall?