MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM EXAM II

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You may not use notes, books, etc. Only the exam paper, a pencil or pen, and a basic or scientific calculator may be kept on your desk during the test.

Good luck!

Problem 1. Let A be the 3×3 matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Find A^{-1} and $A^{-1}A^T$ or show they do not exist. Use your answer to solve the following linear system:

$$\begin{aligned} x_1 + x_3 &= 1, \\ x_2 &= 1, \\ x_1 + x_2 + 2x_3 &= 1. \end{aligned}$$

Solving it in a different way will not receive partial credit.

Problem 2. Consider the following matrices:

$$A = \begin{bmatrix} 4 & 2 & -13 \\ 2 & 1 & -7 \\ 3 & 2 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} -4 & -2 & 13 \\ 0 & -1 & 7 \\ 0 & 0 & -4 \end{bmatrix}.$$

(1) Compute $\det A$. Is A invertible? Explain your answer.
(2) Compute $\det(A + B)$. Is $A + B$ invertible? Explain your answer.

Problem 3. Let

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix}.$$

(1) Find the reduced row-echelon form of A.
(2) What is the rank $\mathrm{rank}(A)$ of A? What is the dimension of the null space $\mathrm{Null}(A)$ of A?
(3) Find a basis of the column space $\mathrm{Col}(A)$ of A.

Problem 4. Consider the initial value problem:

$$y'' + 4y' - 5y = 0, \qquad y(0) = 1, \qquad y'(0) = 2.$$

Will $y(x)$ cross the x axis at some $x \geq 0$? If yes, find all such x.