Math 5615H **Homework 3**

Posted: 12:30 a.m., 09/20, modified on 09/24; due: Friday, 09/26/2014

The problem set is due at the beginning of the class on Friday.

Reading: Chapter 2: pages 24-37

Problems:

1. Suppose a set A is infinite and B is countable. Show that $A \cup B \sim A$. *Hint*: Start with selecting a countable subset in A.

2. Reproduce and complete the argument we had in class (see also a very sketchy note in the text after Theorem 2.14) to show that $|[0,1)| = 2^{\aleph_0}$, where by definition $|A|$ is the cardinality of A, $\aleph_0 := |\mathbb{N}|$, and $2^{|A|} := |P(A)|$, the cardinality of the set $P(A)$ of all subsets of A, for any set A.

3. Show that $\mathbf{c} = 2^{\aleph_0}$, where by definition $\mathbf{c} := |\mathbb{R}|$.

4. If $|A_n| = \mathbf{c}$ for all $n \geq 1$ under the notation of the previous problem, then the countable disjoint union $\coprod_{n \geq 1} A_n$ has the same cardinality $\mathbf{c}$.

5. What is the cardinality of the irrationals?

6. Prove that the following sets are open:

(1) the first quadrant $\{(x, y) \in \mathbb{R}^2 \mid x > 0 \text{ and } y > 0\}$;
(2) any subset of the discrete metric space.

7. Find an infinite collection of distinct open sets in $\mathbb{R}$ whose intersection is a nonempty open set. (Thus infinite intersections of open sets may or may not be open.)

8. Show that $\mathbb{Q}$ as a subset of $\mathbb{R}$ is neither open, nor closed.

9. Show that the closure of set A in a metric space is the intersection of all the closed sets which contain A.