Math 5615H **Homework 4**

Posted: 11:30 a.m., 09/27; modified: 10/01; due: Friday, 10/3/2014

The problem set is due at the beginning of the class on Friday.

Reading: Chapter 2: pages 35-43.

Problems:

1. Let A be a subset of a metric space X and let x_0 be an isolated point of A. Show that x_0 is in the boundary of A if and only if x_0 is a limit point of A^c.

2. Let X be a metric space. If $A \subset X$ has the property that every infinite subset of A has a limit point in A, show that for any open covering there exists a countable or finite subcovering.

3. Let A be a subset of a metric space X. Show that $\overline{A} = A \cup \partial A$, where $\overline{A}$ is the closure and ∂A is the boundary of A.

4. Show that in a discrete metric space, each subset is open and closed.

5. A metric space X is called *separable* if there exists a countable subset of X which is dense in X. Show that if X is compact, then it is separable. *Hint*: For each fixed $n \in \mathbb{N}$, consider the covering of X by the open balls of radius $1/n$ centered at every point of X.

6. Show that the (open) unit ball $B_1(0)$ about the origin 0 in $\mathbb{R}^k$ with the "taxicab" metric is convex.

7. Show that $\mathbb{R}^k$ is separable in the sense of Problem 5 above.

8. Show that any convex subset of $\mathbb{R}^k$ is connected.

9. Show that the Cantor set consists of all the numbers in the closed interval $[0, 1]$ whose ternary expansion has only 0's and 2's and may end in infinitely many 2's.