MATH 5615H: HONORS ANALYSIS
SAMPLE FINAL EXAM (PART I)
NOW, WITH SELECTED SOLUTIONS

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You may not use a calculator, notes, books, etc. Only the exam paper, scratch paper, and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!

Problem 1. Let x_1 be a real number, $x_1 > 1$, and let $x_{n+1} = 2 - 1/x_n$ for $n \in \mathbb{N}$. Show that the sequence $\{x_n\}$ is monotone and bounded and find its limit.

Problem 2. Assume that $f(x)$ is defined a real-valued for $x > 0$. Consider two statements:

(1) For every $m \in \mathbb{N}$, $x > 1/m$ implies $f(x) < 1/m$.
(2) $x > 0$ implies $f(x) \leq 0$.

Prove that (1) implies (2).

Problem 3. Show that for any sequence $\{a_n\}$ of real numbers,

$$\liminf_{n\to\infty} a_n \leq \limsup_{n\to\infty} a_n.$$

You may use any definition of the upper and lower limits, also known as limit superior and limit inferior.

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