Math 5616H **Homework 7**

Posted: 3/23; Updated: 3/24; Due: Friday, 3/27/2015

The problem set is due at the beginning of the class on Friday.

Reading: Chapter 9: pp. 221-228.

Problem 1. Let $U \subset \mathbb{R}^n$ be an open set and $f : U \to \mathbb{R}$. The function f is said to be in $C^k(U)$, if f has all continuous partial derivatives up to and including order k on U. Let $l \leq k$ and $i_1, i_2, \ldots, i_l$ be a collection of integers between 1 and l. Under these conditions, show that for any permutation $\sigma \in S_l$, we have

$$D_{i_1,\ldots,i_l} f = D_{\sigma(i_1),\ldots,\sigma(i_l)} f.$$

Problem 2. Let

$$f(x,y) = \begin{cases} 0 & \text{when } (x,y) = (0,0), \\ (x^2+y^2)\sin\left(\frac{1}{\sqrt{x^2+y^2}}\right) & \text{otherwise.} \end{cases}$$

Show that $D_1 f$ and $D_2 f$ exist everywhere but are not continuous at the origin.

Problem 3. Let $f : \mathbb{R}^3 \setminus \{(0,0,0)\} \to \mathbb{R}^3 \setminus \{(0,0,0)\}$ be given by

$$f(x,y,z) = \left(\frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \right).$$

Show that f is locally invertible at every point in $\mathbb{R}^3 \setminus \{(0,0,0)\}$. Find an explicit formula for f^{-1}.

Problem 4. Consider the equations

$$ab^2 + cde + a^2 d = 3 \qquad \text{and} \qquad ace^3 + 2bd - b^2 e^2 = 2.$$

Determine for which pairs of variables the Implict Function Theorem implies that the two equations can be uniquely solved for in terms of the other three near the point $(a,b,c,d,e) = (1,1,1,1,1)$.

Problem 5. Define a map from $\mathbb{R}^2$ to itself by setting

$$F(x,y) = (\sin x \cos y + \cos x \sin y, \cos x \cos y - \sin x \sin y).$$

Does there exist a point (x_0, y_0) such that F is locally invertible in a neighborhood of $F(x_0, y_0)$?