Special issue in memory of

Karl Walter Gruenberg
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This issue of Journal of Algebra is dedicated to Karl Gruenberg in his memory. We offer some thoughts about his life and work.

Karl was born in Vienna and initially went to school there. In March 1938, when he was 9, German troops invaded Austria and in the wake of Kristallnacht (9 November 1938) the British government agreed to accept Jewish children who were to be transported by train from certain parts of Europe so as to escape the Nazi terror. The children would be placed in homes in Britain. Karl left Vienna on one of these trains of the Kindertransport in March 1939. On arriving in England he was quite unhappy, but the situation improved when his mother was able to join him a while later. He attended Shaftesbury Grammar School in Dorset at first, and then in 1943 Karl and his mother moved to London where he attended Kilburn Grammar School. He went on to win a scholarship to Magdalene College, Cambridge, going there in 1947 and obtaining his BA in 1950, having taken parts I, II and III of the Mathematical Tripos. At this point he became a student of Philip Hall, receiving his Ph.D. in 1954. Prior to this, in 1953, he obtained his first (temporary) appointment as an Assistant Lecturer at Queen Mary College, University of London, where Kurt Hirsch was gradually building up great strength in algebra. He spent the years 1955-7 at Harvard and Princeton, and then from 1957 onwards Queen Mary College became his permanent mathematical home when he was appointed Lecturer. He became Reader in 1961 and Professor in 1967, serving as Head of the Pure Mathematics Department from 1973 to 1978. He retired from these duties in 1993, becoming Professor Emeritus.

In his mathematical work, Karl always remained motivated by group theory, but the emphasis

changed as the years went by. His work was characterized by the wide range of techniques he drew from other areas - homological algebra, commutative algebra, number theory - as well as an alertness to applications in different fields. He was to the fore in developing extraordinary new ideas, with the result that others sometimes found his approach hard to absorb, because it was unfamiliar. His work demonstrates impeccable algebraic technique, often in difficult areas, and this also made it hard on occasion to absorb, in spite of his insistence on elegance, good organization and completeness of explanation. Karl was always excited by the new things that were happening mathematically, and was particularly supportive when they came from younger people. He took the view that to keep up with things in mathematics you must keep moving forward.

Apart from some early papers written as a student, Karl's early work had to do with the Engel structure of infinite groups, in which he had become expert. His first two papers from the 1950s on this topic remain among his most frequently cited works. However, the direction of his research was soon to change as he took an interest in the recently developed homological methods, always with an eye to how they might be applied to groups.

In 1960 Karl introduced a resolution of the trivial module for an integral group ring which he called a 'resolution by relations', since it depends on a presentation of the group. The resolution is now known as the 'Gruenberg resolution' and it has had a profound impact on group cohomology, appearing in many books on the subject. He used these ideas in 1967 to give a new treatment of extension theory for groups, and again in 2003 to give a group-theoretical interpretation of higher cohomology groups.

Karl's book, 'Cohomological topics in group theory', dating from 1970, presents an exposition of the basic theory of group cohomology using the Gruenberg resolution and his approach to extensions as well as some other novelties. His approach retains its validity today as one of the best ways to introduce group cohomology. The book is also a remarkable compendium of applications of cohomology to group-theoretic questions, many of which are still not available elsewhere in book form. In the latter part of the book Karl described the start of a new development of extension categories, requiring facts from integral representation theory. He was to take both these topics much further during the following years.

In 1970 Karl introduced the term 'relation module' and initiated the study of relations modules of groups as integral representations (Over a field, something had already been done in this direction by Gaschütz.) He had the idea to apply localization techniques and also the cancellation techniques which had been developed earlier by Swan, Heller, Jacobinski and others to establish certain invariants of the group, and of its presentations. These ideas were explained in his book on relation modules, which remains the most complete account of this subject. It was the start of an area of research which has been active ever since.

In work with Klaus Roggenkamp which appeared in 1975, Karl realized the significance of the connectivity of the prime graph of a group in connection with the decomposition of the augmentation ideal and relation modules and settled when they decompose in the soluble case. The prime graph was in fact defined in an unpublished paper with Kegel, and is sometimes also known as the Gruenberg-Kegel graph. They drew attention to a structure about which many tens of papers have since been written.

During the 1970s and early 1980s Karl continued the work on extension categories (in part with Roggenkamp) which he had begun earlier. Also in the 1980s Karl worked on questions of minimality of free resolutions over integral group rings, obtaining some of the most incisive results in this area.

His most substantial collaboration was yet to come: he wrote 11 publications with Al Weiss, the last of which appears in this volume. These papers have to do with the Galois module structure of

various modules constructed from extensions of number fields. They use techniques involving integral representation theory and group cohomology - the subjects which had been central to his work throughout his career.

Karl was a talented teacher, both in the classroom setting and also as the supervisor of 16 Ph.D. students. He was not overly pedantic, but he did insist on the highest standard of clear and careful explanation as well as elegant organisation, He required this of others as well as demonstrating these qualities in his own presentations. He was aware of the importance of pace when giving a talk and took delight in introducing elements of wit and humour. We remember, for example, his evident pleasure in exploiting the double meaning in the term 'generation gap' which he had introduced to denote a certain mathematical quantity to draw a laugh from his audience. As a student, one really could not have done better than have him as a Ph.D. adviser. For young people finding their way in the mathematical world he provided a firm mathematical grounding, inspiration, and also help with the initial steps in a mathematical career. He would never fail, for example, to introduce his students to the distinguished mathematical visitors who came to Queen Mary, in many cases to the subsequent benefit of the student.

Karl is dearly remembered by many people around the world. This has to do with his own great hospitality, his interest in other people, his appreciation of social interaction, and the pleasure it was to be with him. We remember his skill and poise in conversation, his wit, and the fact that he was never at a loss for something to say. He was widely interested in all things cultural, but most notably in the theatre (he acted as a student), music, art and architecture. He loved to travel and spent lengthy periods as a visitor in mathematics departments around the world. This started in 1955 with a year at Harvard, followed by a year at Princeton starting in 1956. He spent a year at the University of Michigan (1961-2) and at Cornell (1966-7) and this trend continued: in the 1970s he visited Vancouver, Urbana, Michigan and Canberra as well as many other places. Karl was acquainted with very many people from all over the world, whom he visited, and whom he was more than pleased to welcome when they in turn visited Queen Mary. He had very many friends and is greatly missed.

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