Mathematics 8302, Spring 2002
Manifolds and Topology

Lectures: MWF 1:25-2:15
Texts:

(1) J.J. Rotman, An Introduction to Algebraic Topology, Springer Graduate Texts in Math. 119, 4th printing 1998,
(2) T. Aubin, A course in differential geometry, A.M.S. 2001.

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Office hours: MWF 11:15-12:05 and probably 3:35-4:25 or by appointment.

1. **Course Assessment.** Your grade will be determined by your performance on homework, quizzes given in class and a final exam. Starting on February 4, I will take in homework from you each Monday. Homework may be given to me during class, and I will also accept it if you put it in my mailbox before 4pm on Monday. I will not accept late homework. There will be about 13 sets of homework altogether during the quarter. Every other Monday there will be a 30 min. quiz in class on the subject matter of the homework due that day and on the previous Monday. There will be 6 quizzes altogether, on 2/11/02, 2/25/02, 3/11/02, 4/1/02, 4/15/02 and 4/29/02. There will be no make-up quizzes. We will finish with a final exam on all the topics covered. If you approve, I will make it an optional take-home exam in the same manner as last semester. Each quiz will count 6%, the homeworks will count 48%, and the final exam will count 16%. Only the best 12 homework scores will be used, and if a quiz is missed with good reason I will transfer available credit from that quiz to the remaining quizzes.

2. **Syllabus.** Last semester we got as far as the long exact sequence in homology associated to a short exact sequence of chain complexes, Theorem 5.6 in Rotman's book. We pick up from there and do the relative homology groups (Theorem 5.8) and excision and Mayer-Vietoris sequence in the context of simplicial homology (Theorems 7.16 and 7.17). I intend to assume, but not prove, the following theorem, and this will enable us to work with simplicial homology where it is more convenient to do so:

THEOREM (7.22 and 9.8 of Rotman). *Let K be a finite simplicial complex. Then the complexes of singular chains, ordered chains and oriented chains are all chain homotopy equivalent. Hence the homology groups of these complexes are isomorphic.*

From there we do applications to spheres and $\mathbb{R}^n$ (pages 109-111), the degree of a map, antipodal maps, hairy ball and Borsuk-Ulam theorems (pages 119-125), the Euler characteristic (pages 145-6, 152) and the Lefschetz fixed point theorem (pages 247-252).

We also need to classify compact surfaces, which is given a brief treatment on page 195, and for which I will probably hand out supplementary material.

After all this we have to do differentiable manifolds: essentially the material of chapters 0 - 3 of Aubin's book plus some more about curvature. There is rather a lot to do, and at times I feel I may have to go rather quickly!

3. **Other books** The following are on reserve in the library:

T. Aubin, A course in differential geometry, A.M.S. 2001, QA 641 A795 2001
L. Conlon, Differentiable manifolds, Birkhaüser 2001, QA 614.3 C66 2001
V. Guillemin and A. Pollack, Differential topology, Prentice-Hall 1974, QA 613.6.G84
S. Lang: Fundamentals of differential geometry, Springer Graduate Texts in Mathematics 191 (1999), QA 641 L33 1999

Here is another book:

F.W. Warner, Foundations of differentiable manifolds and Lie groups, Springer Graduate Texts in Mathematics 94, QA 614.3.W37 (1983)

4. **Expectations of your work.** You may discuss homework problems with other students, indeed I encourage you to do this; but I would like each person to write out their own homework as an independent effort. I expect the final exam to be entirely your own work, done without any collaboration.

As concerns your written style, I expect your homework to contain full written explanations of your arguments. These should be written in English sentences (recall that sentences start with a capital letter, contain a verb and finish with a period!), and read smoothly as English. If some portion of argument is missing from what you write, you will not get credit by explaining afterwards that you knew it really but you just omitted to write it down. I expect that you all will come with some experience of writing mathematical arguments in this fashion.