# Math 1031 Final Exam 

December 15, 2000 3 hours

Closed book, closed notes. Simple calculators are allowed. Work all problems.
The first 18 problems are multiple choice. Please mark the correct answer with a number 2 pencil on the separate answer sheet provided. Only the correct answer will receive credit -- there is no partial credit.

Problems 19-24 are free response. Please work the problem in the space provided and show all work. Partial credit will be given. A correct answer may receive less than full credit if the work is incomplete or incorrect.

Please write your name on every page.

| Problem | Score | Possible |
| :---: | ---: | ---: |
| $1-18$ |  | 144 |
| 19 |  | 20 |
| 20 |  | 20 |
| 21 |  | 32 |
| 22 |  | 30 |
| 23 |  | 24 |
| 24 |  | 30 |
| Total |  | 300 |

(8) 1. Suppose that you flip a fair coin four times and then roll a fair die three times. What is the probability that you will get the sequence $\mathrm{H}-\mathrm{H}-\mathrm{H}-\mathrm{H}-3-2-1$ ?
a. $\frac{1}{2^{4} \cdot 6 \cdot 5 \cdot 4}$
b. $\frac{1}{2^{4} \cdot 6^{3}}$
c. $\frac{1}{2^{4}}+\frac{1}{6^{3}}$
d. $\frac{1}{4 \cdot 2 \cdot 3 \cdot 6}$
e. $\frac{1}{4 \cdot 2}+\frac{1}{3 \cdot 6}$
(8) 2. Suppose you invest $\$ 5000$ compounded continuously at a rate of $7 \%$ annual interest.

After $T$ years you have $\$ 10,000$. Which correctly expresses $T$ ?
a. $\ln \left(\frac{2}{0.07}\right)$
b. $\frac{\ln 2}{0.07}$
c. $\frac{\ln 0.5}{0.07}$
d. $0.07 \cdot \ln 2$
e. none of the above
(8) 3. What is the solution of $\frac{1}{2 y+3}=\frac{4}{5 y+6}$ ?
a. There is no solution.
b. $y=\frac{1}{2}$
c. $y=-2$
d. $y=2$
e. none of the above
(8) 4. What is the solution of the following system?

$$
\begin{aligned}
& 5 x+3 y=3 \\
& 3 x+2 y=1
\end{aligned}
$$

a. There is no solution.
b. $(x, y)=\left(\frac{2}{3},-\frac{1}{2}\right)$
c. $(x, y)=\left(\frac{2}{3},-\frac{1}{9}\right)$
d. $(x, y)=(3,-4)$
e. none of the above
(8) 5. What is the slope of the line passing through the points $(2,-1)$ and $(-1,1)$ ?
a. -2
b. -1
c. $-\frac{2}{3}$
d. $\frac{2}{3}$
e. 1
(8) 6. Which answer describes the solution of $|1-3 x|>1$ ?
a. $x>\frac{2}{3}$
b. $x<\frac{2}{3}$
c. $0<x<\frac{2}{3}$
d. $x<0$ or $x>\frac{2}{3}$
e. $x=0$
(8) 7. A total of $\$ 30,000$ was invested in two funds paying $6 \%$ and $7 \%$ simple annual interest. If $\$ 2050$ in interest was earned after one year, how much was invested in each fund?
a. $\$ 25,000 @ 6 \%$ and $\$ 5,000 @ 7 \%$
b. $\$ 20,000 @ 6 \%$ and $\$ 10,000$ @ 7\%
c. $\$ 15,000 @ 6 \%$ and $\$ 15,000 @ 7 \%$
d. $\$ 10,000 @ 6 \%$ and $\$ 10,000 @ 7 \%$
e. $\$ 5,000 @ 6 \%$ and $\$ 25,000 @ 7 \%$
(8) 8. What are the real solutions of $x^{2}+4 x-12$ ?
a. No real solution.
b. $x=2$
c. $x=2$ and $x=-6$
d. $x=6$ and $x=-2$
e. $x=6$
(8) 9. You flip three fair coins. What is the probability that all three show heads, given that at least one is a head?
a. $\frac{1}{8}$
b. $\frac{1}{7}$
c. $\frac{1}{6}$
d. $\frac{1}{4}$
e. $\frac{1}{3}$
(8) 10. Which answer is equal to $\ln \left(12^{2}\right)$ ?
a. $\ln (12) \cdot \ln (12)$
b. $(\ln (12))^{2}$
c. $\ln (2) \cdot \ln (12)$
d. $12 e^{2}$
e. $4 \ln (2)+2 \ln (3)$
(8) 11. What is the maximum value of the function $f(x)=2-6 x-3 x^{2}$ ?
a. -2
b. -1
c. 2
d. 3
e. 5
(8) 12. Suppose that $g$ is a function whose graph contains the following points $(1,2),(2,3)$, and $(3,2)$. What is $g^{-1}(2)$ ?
a. 1
b. 2
c. 3
d. $g^{-1}$ does not exist.
e. none of the above
(8) 13. You have a one dollar bill, a five dollar bill, and a ten dollar bill. How many ways can you hand out all the bills to six people, if anyone can get any number of bills?
a. $P(6,3)$
b. $C(6,3)$
c. $6^{3}$
d. $3^{6}$
e. none of the above
(8) 14. What is the solution of $x^{2}<3 x+4$ ?
a. $x<2$
b. $x>4$
c. $x<-1$
d. $-1<x<4$
e. $x>4$ and $x<-1$
(8) 15. If $f(x)=\frac{1}{x^{2}}$ and $g(x)=x^{2}-1$, what is $g(f(x))$ ?
a. $\frac{x^{2}-1}{x^{2}}$
b. $x^{2}-\frac{1}{x^{2}}$
c. $\frac{1}{\left(x^{2}-1\right)^{2}}$
d. $\frac{1}{x^{4}}-1$
e. 0
(8) 16. What is the equation of the line with slope 2 passing through the point $(-3,-5)$ ?
a. $y=2 x+1$
b. $y=2 x-5$
c. $y=2 x+5$
d. $y=2 x-3$
e. $y=2 x+3$
(8) 17. Which of the following parabolas has vertex $(2,-3)$ ?
a. $y=x^{2}-4 x+1$
b. $y=x^{2}-4 x+7$
c. $y=x^{2}-2 x-3$
d. $y=x^{2}+4 x+1$
e. $y=x^{2}+4 x+7$
(8) 18. You flip a fair coin six times. What is the probability of getting at least two heads?
a. $\frac{3}{32}$
b. $\frac{7}{64}$
c. $\frac{1}{3}$
d. $\frac{57}{64}$
e. $\frac{29}{32}$
(20)19. Find the inverse of the function $f(x)=\frac{x}{3 x-2}$.
(20)20. Consider the circle equation $9 x^{2}+9 y^{2}-6 x+24 y+1=0$.
(15) a. Find the standard form of the equation.
(5) b. Find the radius and center, and sketch the graph.
(32)21. Suppose that you draw 3 cards from a standard deck of 52 cards. What is the probability of getting:
(8) a. exactly one ace?
(8) b. exactly two aces?
(8) c. no aces?
(8) d. exactly three clubs?
(30)22. Consider the following system of equations:

$$
\begin{aligned}
x+6 y+4 z & =9 \\
8 x+7 y+z & =11 \\
x+y+z & =0
\end{aligned}
$$

(20) a. Put these equations into row-echelon form.
(10) b. Back-substitute to find the solution.
(24)23. You decide to play the following game. You draw one card from a standard deck of 52 cards. If you draw an ace, you win $\$ 20$. If you draw a face card, you win $\$ 10$. If you draw any other card, you lose the amount in dollars shown on the card. (I.e., if you draw a 7, you lose \$7.) What is the expected value of this game?
(30)24. Solve for x :
(10) a. $x^{4}-3 x^{2}-4=0$
(10) b. $2^{2 x}-3 \cdot 2^{x}-4=0$
(10) c. $\log _{2} x+\log _{2}(x-3)=2$

