Math 8201 **Homework 12** **PJW**

Date due: Friday December 5, 2008. There will be a quiz on this date. Hand in the 8 starred questions.

Section 6.2 6.18, 6.20*, 6.21, 6.22*, 6.23(iv)*, 6.23(v)*, 6.23(vi)*, 6.23(vii)*, 6.25, 6.26, 6.27, 6.30*

FFF Let R be an integral domain with quotient field F and let $P(x)$ be a monic polynomial in $R[x]$. Assume that $p(x) = a(x)b(x)$ where $a(x)$ and $b(x)$ are monic polynomials in $f[x]$ of smaller degree than $p(x)$. Prove that if $a(x) \notin R[x]$ then R is not a UFD. Deduce that $\mathbb{Z}[2\sqrt{2}]$ is not a UFD.

GGG* Prove that if $f(x)$ and $g(x)$ are polynomials with rational coefficients whose product $f(x)g(x)$ has integer coefficients, then the product of any coefficient of $g(x)$ with any coefficient of $f(x)$ is an integer.

HHH Let $R = \mathbb{Z} + x\mathbb{Q}[x] \subset \mathbb{Q}[x]$ be the set of polynomials in x with rational coefficients whose constant term is an integer.

(a) Prove that R is an integral domain and its units are ± 1.

(b) Show that the irreducibles in R are $\pm p$ where p is a prime in $\mathbb{Z}$ ad the polynomials $f(x)$ that are irreducible in $\mathbb{Q}[x]$ and have constant term ± 1. Prove that these irreducibles are prime in R.

(c) Show that x cannot be written as the product of irreducibles in R (in particular, x is not irreducible) and conclude that R is not a UFD.

(d) Show that x is not a prime in R and describe the quotient ring $R/(x)$.