Math 8202 **Homework 11** **PJW**

Date due: Wednesday April 29, 2009. Hand in only the 5 starred questions.

Homework 10 is in fact due on Wednesday April 22. We will hold Quiz 5 (on the material of HWs 9 and 10) on April 27 and Quiz 6 on May 4. This is because the Algebra preliminary exam is on April 20.

Section 9.1 page 663 no. 9.2 (you may need to read what a (p)-primary module is – one such that every element is annihilated by a power of p), 9.3

Section 9.4 page 694 9.47, 9.48

OO* Two linear transformations $S, T : V \to V$ where V is a vector space are said to be *similar* if there is an invertible linear transformation $A : V \to V$ so that $T = ASA^{-1}$. Prove that similar linear transformations have the same characteristic polynomial and the same minimal polynomial.

PP* Prove that two 2×2 matrices over a field F which are not scalar matrices are similar if and only if they have the same characteristic polynomial.

QQ Prove that two 3×3 matrices over a field F are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this assertion for 4×4 matrices.

RR* Find the rational canonical forms of

$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} c & 0 & -1 \\ 0 & c & 1 \\ -1 & 1 & c \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 42 & 465 & 15 & -30 \\ -420 & -463 & -15 & 30 \\ 840 & 930 & 32 & -60 \\ -140 & -155 & -5 & 12 \end{pmatrix}.$$

SS Find all similarity classes of 6×6 matrices over $\mathbb{C}$ with characteristic polynomial $(x^4 - 1)(x^2 - 1)$.

TT Find all similarity classes of 3×3 matrices A over $\mathbb{Q}$, and also over $\mathbb{F}_2$ satisfying $A^6 = I$. Do the same for 4×4 matrices B satisfying $B^{20} = I$.

UU* Find all similarity classes of 6×6 matrices over $\mathbb{Q}$ for which the minimal polynomial is $(x + 2)^2(x - 1)$.

VV* Determine up to similarity all 2×2 matrices with entries in $\mathbb{Q}$ of precise order 4 (multiplicatively, of course). Do the same if the matrix has entries from $\mathbb{C}$.

WW Determine representatives for the conjugacy classes for $GL_3(\mathbb{F}_2)$.

XX Let V be a finite dimensional vector space over $\mathbb{Q}$ and suppose T is a nonsingular linear transformation of V such that $T^{-1} = T^2 + T$. Prove that the dimension of V is divisible by 3. If the dimension of V is precisely 3 prove that all such transformations T are similar.