Math 8202 **Homework 12** **PJW**

Do not hand in this homework. Quiz 6 will be held on Monday May 4 on the topics of Homeworks 11 and 12.

Section 9.3 9.37

YY Find the Jordan canonical forms of the following matrices and find a matrix P which conjugates the matrix to its Jordan canonical form.

$$\begin{pmatrix} 9 & 4 & 5 \\ -4 & 0 & -3 \\ -6 & -4 & -2 \end{pmatrix} \quad \begin{pmatrix} 5 & 4 & 1 \\ -1 & 0 & 0 \\ -3 & -4 & 1 \end{pmatrix} \quad \begin{pmatrix} 3 & 4 & 2 \\ -2 & -3 & -1 \\ -4 & -4 & -3 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

ZZ Determine which of the following matrices are similar:

$$\begin{pmatrix} -1 & 4 & -4 \\ 2 & -1 & 3 \\ 0 & -4 & 3 \end{pmatrix} \quad \begin{pmatrix} -3 & -4 & 0 \\ 2 & 3 & 0 \\ 8 & 8 & 1 \end{pmatrix} \quad \begin{pmatrix} -3 & 2 & -4 \\ 2 & 1 & 0 \\ 3 & -1 & 3 \end{pmatrix} \quad \begin{pmatrix} -1 & 4 & -4 \\ 0 & -3 & 2 \\ 0 & -4 & 3 \end{pmatrix}$$

AAAA Prove that for every matrix A, A is similar to its transpose A^t.

BBBB Show that if A is a matrix for which $A^2 = A$ then A is similar to a diagonal matrix which has only 0s and 1s along the diagonal.

CCCC (Fall 2001 qn. 7) Let A and B be endomorphisms of a complex vector space V which commute with each other. We say that V is *indecomposable* if it cannot be written $V = V_1 \oplus V_2$ where V_1 and V_2 are non-zero subspaces, each of which is mapped to itself by both A and B.

(a) (8) Suppose V is indecomposable. Show that A has only one eigenvalue.

(b) (7) Suppose now that there is no subspace V_1 of V which is mapped to itself by both A and B and with $0 \neq V_1 \neq V$. Show that $\dim_{\mathbb{C}} V = 1$

DDDD (Spring 2001 qn. 3) Let $T : \mathbb{C}^n \to \mathbb{C}^n$ be an invertible linear transformation of finite order. Show that $\mathbb{C}^n$ has a basis with respect to which the matrix of T is diagonal.

EEEE (Spring 2002 qn. 6) Let V be a finite-dimensional complex vector space, and let $T : V \to V$ be a linear transformation.

(a) (6%) Give a condition on the minimal polynomial which is necessary and sufficient for T to be diagonalizable. Prove that your answer is correct.

(b) (6%) If the minimal polynomial of T is equal to the characteristic polynomial of T, then what can we say about the Jordan canonical form of T? Be as specific as possible, and prove that your answer is correct.

(c) (6%) Prove that if $W \subseteq V$ is an invariant subspace (thus $T(W) \subseteq W$) and T is diagonalizable, then the restriction $T|_W$ (considered as a linear transformation from W to itself), also is diagonalizable.