

Review for Math2574 Exam 2 (Thursday March 31 in recitation)

The exam is on sections 3.1-3.6, 4.1-4.5, 4.7 from the book.

The questions below are intended to help, not to be a list of things which, by implication, you should or should not know. I have tried to find a question for each important topic, but I may have missed some out. You should still know topics if I have missed them out.

Section 3.1: 12 and others similar.

Section 3.2: 11

Section 3.3: 16

Section 3.4: 9

Section 3.5: 13

Section 3.6: 5, 12, 18

Section 4.1: 17, 20

Section 4.2: 4, 5, 16

Section 4.3: 17, 18, 26, 32, 33, 35

Section 4.4: 5, 7, 13, 21

Section 4.5: 6, 14, 19, 25, 27, 28, 30, 31, 32 (some of these were on Ass. 8)

Section 4.7: 3, 10, 12

There are fewer questions from the earlier sections because when you do the questions from the later sections you also get practice with the earlier sections.

Some True/False questions: Determine whether each of the following statements is true or false, giving brief reasons or a counterexample. Let A be an $m \times n$ matrix.

- (a) If, for some vector b , there is exactly one solution to $Ax = b$ then the nullspace of A must be zero.
- (b) If A has zero nullspace then every equation $Ax = b$ always has exactly one solution.
- (c) If, for all b , $Ax = b$ has at most one solution then $m \geq n$.
- (d) If, for all b , $Ax = b$ has at most one solution then $m \leq n$.
- (e) If $n \geq m$ then, for all b , $Ax = b$ has at least one solution.
- (f) If $n \geq m$ then $Ax = 0$ has infinitely many solutions.
- (g) If $Ax = 0$ has a unique solution then the columns of A span \mathbb{R}^m .
- (h) If $Ax = 0$ has a unique solution then the columns of A are independent.
- (i) If $Ax = 0$ has a unique solution then the rank of A is m .
- (j) If $Ax = 0$ has a unique solution then the rank of A is n .
- (k) $Ax = b$ has a unique solution for all b if and only if $m = n$ and $\det A \neq 0$.
- (l) There is a 3×3 matrix whose nullspace is the space spanned by $(1, 1, 1)$.
- (m) There is a 3×3 matrix whose nullspace is the space spanned by $(1, 1, 1)$ and whose row space is the space spanned by $(1, -1, 0)$.
- (n) Every subspace of \mathbb{R}^n can be spanned by at most n vectors.