Math 3592H Honors Mathematics I Fall Semester 2017

Assignment 5 - Due Thursday 10/12/2017

Read: Hubbard and Hubbard Section 1.6.

Exercises:
Hand in only the exercises which have stars by them.

Section 0.7: 4, 9, 12(a)(ii)*, 13, 14.
Section 1.6: 1, 2*, 3*, 4, 5, 6, 7 (look at the hint below).

Extra questions:
1. Which of the following sequences of complex numbers tend to a limit? Justify your answer.
(i) * $a_n = \dfrac{n+i}{2n-3i+5}$, (ii) $b_n = \dfrac{(2-i)^n}{e^n}$, (iii) $c_n = \dfrac{n^2+5i}{14ie^n}$

2. Which of the following series of complex numbers converge? Justify your answer.
(i) * $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(1+i)^{2n}+(1-i)^n}$ (ii) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+i}$ (iii) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+i}$

Comments:
With the homework questions this week, one of the things we do in Section 1.6 is learn about the fundamental theorem of algebra, which is about complex numbers, so I have put in some questions about complex numbers to make sure that we can all do them. The extra questions have to do with limits, and these are defined for the complex numbers by regarding them as a copy of 2-dimensional space over the real numbers. A sequence converges if and only if in each coordinate the terms converge (see 1.5.13, 1.5.22) and a similar thing is true for series. I am hoping that you can probably figure these things out from what you already know, without me teaching it.

Some of the material at the moment is quite sophisticated we are taking a long time over these sections. Although I have listed questions 4, 5, 6, 7 from section 1.6, we probably will not have covered the material in time this week. If that is the case, don't do the questions.

Hint for question 1.6.7: Consider the function g(x) = f(x) - mx. Where it has a minimum the derivative is zero. Get an expression for that derivative to see that it will produce an answer to the question. You have to show that the minimum cannot occur at either a or b, and this is the hardest part. For this, use the fact that if g is a function for which g'(a) < 0 then for all y in some small open interval (a,a+u) we have g(y) < g(a). The proof of this comes from examining the definition of the limit which appears in defining the derivative.