

Assignment 8 - Due Thursday 11/02/2017

Read: Hubbard and Hubbard Section 1.9. I hope to start on 2.1 during the week.

Exercises:

Hand in only the exercises which have stars by them.

Section 1.9 (page 152): 1.9.2c*

Section 1.10 (pages 156-157): 28b, 29, 30*, 31*, 32, 33

Hint for question 30a in section 1.10: the area of the parallelogram is the length of the cross product of the vectors.

Extra Questions (sorry about the typescript):

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 + xy + 1 \\ y^2 + 2 \end{pmatrix}, \quad g\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u + v \\ 2u \\ v^2 \end{pmatrix}$$

1. Given that find the derivative matrix of the composite function $g \circ f$ at $(x,y) = (1,1)$; also at $(x,y) = (0,0)$.

$$f(t) = \begin{pmatrix} t \\ t^2 - 4 \\ e^{t-2} \end{pmatrix}, \quad -\infty < t < \infty.$$

2. Consider the curve defined parametrically by Let $g(x,y,z)$ be a real-valued differentiable function of three variables. If $a = (2,0,1)$ and

$$\frac{\partial g}{\partial x}(a) = 4, \quad \frac{\partial g}{\partial y}(a) = 2, \quad \frac{\partial g}{\partial z}(a) = 2, \quad \text{find } \frac{d(g \circ f)}{dt} \text{ at } t = 2.$$

3*. Let $z = xy^2$ and suppose that $x = 2u + 3v$. Assume also that y is a function of u and v

with the properties that when $(u,v) = (2,1)$ then $y = -1$, $\frac{\partial y}{\partial u} = 5$ and $\frac{\partial y}{\partial v} = -2$. Find

$$\frac{\partial z}{\partial u} \text{ and } \frac{\partial z}{\partial v} \text{ when } (u,v) = (2,1).$$

4*. Show that for a differentiable real-valued function $g(x,y)$,

$$\frac{dg(x, x)}{dx} = \frac{\partial g}{\partial x}(x, x) + \frac{\partial g}{\partial y}(x, x).$$

Apply this equation to the function $g(x,y) = x^y$ and hence compute the derivative of the function $u(x) = x^x$. [Hint: Consider the composite of g with the function $f(x) = (x,x)$.]

Comments:

The main result in this section, Theorem 1.9.8, is important to know, but I do not think we need trouble ourselves much with the proof. The proof depends on Theorem 1.9.1 and here also we don't need to bother. You will not be tested on those proofs. What is more important in section 1.9 is to see the type of strange behavior that can occur in the examples.