Math 8201 **Homework 9** **PJW**
Date due: November 13, 2017.

Hand in only the starred questions.

Section 5.1 1, 2, 4*, 5, 6, 18.
Section 5.4 2, 4, 7*, 10, 11*, 13*, 15, 17, 19.

GG. Let G be the group of *all* isometries of the cube, and let H be the subgroup consisting of rotations which preserve the cube. Let -1 denote the element of G which is the transformation of $\mathbb{R}^3$ given by multiplication by -1.
(a) Show that $G = H \times \langle -1 \rangle$.
(b) Show that if $g \in G$ is any element other than -1 then $G \neq H \times \langle g \rangle$.
(To do this you may need to prove that the center of H is $\{e\}$. Either use the isomorphism with S_4 or note that if you conjugate one rotation by another rotation you get rotation about an axis obtained by applying the second rotation to the axis of the first.)

HH*. (a) Let G be the group of *all* isometries of the tetrahedron, and let H be the subgroup consisting of rotations which preserve the tetrahedron. Determine whether or not $G = H \times K$ for some subgroup K of G.
(b) Let G be the group of *all* isometries of the icosahedron, and let H be the subgroup consisting of rotations which preserve the icosahedron. Determine whether or not $G = H \times K$ for some subgroup K of G.