Review for Math2574 Exam 2 (On Thursday March 28)

The exam is on sections 3.1-3.6, 4.1-4.5, 4.7 from the book.

The questions below are intended to help, not to be a list of things which, by implication, you should or should not know. I have tried to find a question for each important topic, but I may have missed some out. You should still know topics if I have missed them out.

Section 3.1: 12 and others similar.
Section 3.2: 11
Section 3.3: 16
Section 3.4: 9
Section 3.5: 13
Section 3.6: 5, 12, 18
Section 4.1: 17, 20
Section 4.2: 4, 5, 16
Section 4.3: 17, 18, 26, 32, 33, 35
Section 4.4: 5, 7, 13, 21
Section 4.5: 6, 14, 19, 25, 27, 28, 30, 31, 32 (some of these were on Ass. 8)
Section 4.7: 3, 10, 12

There are fewer questions from the earlier sections because when you do the questions from the later sections you also get practice with the earlier sections.

Some True/False questions: Determine whether each of the following statements is true or false, giving brief reasons or a counterexample. Let A be an $m \times n$ matrix.

(a) If, for some vector b, there is exactly one solution to Ax = b then the nullspace of A must be zero.

- (b) If A has zero nullspace then every equation Ax = b always has exactly one solution.
- (c) If, for all b, Ax = b has at most one solution then $m \ge n$.
- (d) If, for all b, Ax = b has at most one solution then $m \le n$.
- (e) If $n \ge m$ then, for all b, Ax = b has at least one solution.
- (f) If $n \ge m$ then Ax = 0 has infinitely many solutions.
- (g) If Ax = 0 has a unique solution then the columns of A span \mathbb{R}^m .
- (h) If Ax = 0 has a unique solution then the columns of A are independent.
- (i) If Ax = 0 has a unique solution then the rank of A is m.
- (j) If Ax = 0 has a unique solution then the rank of A is n.
- (k) Ax = b has a unique solution for all b if and only if m = n and det $A \neq 0$.
- (l) There is a 3×3 matrix whose nullspace is the space spanned by (1, 1, 1).

(m) There is a 3×3 matrix whose nullspace is the space spanned by (1, 1, 1) and whose row space is the space spanned by (1, -1, 0).

(n) Every subspace of \mathbb{R}^n can be spanned by at most *n* vectors.