Math 3593 Practice for exam 3.

You will be allowed to use books, notes and a calculator on this exam.

A Find the area of the plane elliptical region which is the part of the plane $z = 4 - x - 2y$ that lies above the circle $x^2 + y^2 \leq 1$ in the xy-plane.

B Find the surface area of the part of the graph of the function $z = y^2 - x^2$ which lies above the circle $x^2 + y^2 \leq 1$ in the xy-plane.

C Let A be the unit circle $x^2 + y^2 \leq 1$ and let $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation given by $\Phi \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + y \\ x - y \end{pmatrix}$. Find the area of $\Phi(A)$.

D Calculate $\int_B e^{(x^2+y^2+z^2)^{3/2}} dx\, dy\, dz$ where B is the unit ball $x^2 + y^2 + z^2 \leq 1$.

E Find the length of the part of the helical spiral in $\mathbb{R}^3$, specified in cylindrical polar coordinates (r, θ, z) by $r = 2$, $z = 3\theta$, for which $0 \leq \theta \leq 2\pi$.

F Find the area of the plane elliptical region which is the part of the plane $z = 4 - x - 2y$ that lies above the circle $x^2 + y^2 \leq 1$ in the xy-plane.

G Let $\omega(\mathbf{v}, \mathbf{w}) = e^z v_2 w_1 - e^y v_1 w_2 + \sin xv_2 w_3 - \sin xv_3 w_2$. Express $d\omega$ as a linear combination of elementary forms.

H Find a 2-form on $\mathbb{R}^4$ which is zero on the 3-space spanned by the first three coordinate directions.

I Find a 2-form on $\mathbb{R}^4$ which is zero on $w + x + y + z = 0$.

J What is the dimension of the space of 2-forms on $\mathbb{R}^4$ which restrict to zero on the 3-space spanned by the first three coordinate directions.

K Find a 2-form that orients the surface $e^{(x+y)} + e^{\sin z} = 1$ in $\mathbb{R}^3$.

L Let $\omega(\mathbf{v}, \mathbf{w}) = v_1 w_2 - v_2 w_1$, $\psi(\mathbf{v}, \mathbf{w}) = v_1 w_3 - v_3 w_1 + v_3 w_4 - v_4 w_3$. Evaluate $\omega \wedge \psi$ on the list of four vectors which are the transposes of $(1, 2, 3, 4)$, $(4, 3, 2, 1)$, $(1, 0, 1, 0)$, $(0, 0, 1, 1)$.

M Consider the manifold $M \subset \mathbb{R}^3$ specified by $x + 3y^2 + e^{y+z} = 0$ and with parametrization

$$\Phi \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} -3y^2 - e^{y+z} \\ y \\ z \end{pmatrix}$$

where $y \geq 0$. Suppose that Φ is a direct parametrization. ∂M is parametrized by

$$\gamma(z) = \begin{pmatrix} -e^z \\ 0 \\ z \end{pmatrix}.$$

Is ∂M correctly oriented by dx or by $-dx$?