Math 8245 Group Theory Fall Semester 2010

Instructor

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Office Hours: MWF 11:15 - 12:05 or by appointment.

Course Content and Goals

The overall goal of the first semester is to teach a range of topics which go beyond the basic algebra course Math 8201/2 and which should be known by, and be useful to, the well-educated practitioner of group theory. I intend everything we study either to be interesting in its own right in some reasonable sense, or else a result we need to get to something else.

For the first 10 weeks or so of this semester there will be two parallel courses, one addressing the theoretical side of finite groups, the other dealing with computational group theory using the computer system GAP (computational methods being essential these days). The theoretical side will be taught on 2 days each week and on the third class day each week (still to be determined) we will meet in a computer lab. For this you will need an account on the Math Dept computers. No prior programming experience is necessary. We will start by learning the basics of the language which GAP uses, and go on to learn how to do computations with groups given as permutation groups, by presentations, and as matrix groups. As part of this we will learn how the algorithms we use work, and thereby gain some insight into their limitations.

On the theoretical side we will start with some basics of group theory: semidirect products, wreath products, Sylow subgroups of symmetric groups, actions on sets, semidihedral, dihedral and quaternion groups, groups of order the cube of a prime.

We will then consider some more specialized things: stabilizer chain algorithms.; the Todd-Coxeter algorithm for coset enumeration; simplicity of PSL groups (when they are simple); construction and simplicity of the Mathieu groups; groups acting on trees. If we finish that we could study other basic things: nilpotent and solvable groups, for example. We could also start on the work of the second semester.

Second semester: at the moment I think to do group cohomology, but I could teach various things, for instance group representations or representations of algebras (including Auslander-Reiten theory) or representations of the symmetric groups (in arbitrary characteristic) and Schur algebras

Texts and sources

There will be no text to purchase. I will distribute handouts in class describing the theory we are covering and indicate where it can be found in books. We will study GAP using materials which you can download from my home page. I list some books below which will probably be useful.

Books on theoretical aspects

DJS Robinson, A course in the theory of groups, 2nd edition, Springer-Verlag, ISBN 0387944613 (most useful for general group theory)
J. Rotman, An introduction to the theory of groups (general group theory, Mathieu groups).
M. Isaacs, Finite Group Theory, AMS 2008, QA177 I835 2008 (a rather deep account).
O. Bogopolsky, Introduction to group theory, European Math. Soc. 2008, QA174.2.B64 2008 (general group theory, Mathieu groups, trees)
W. Dicks and M. Dunwoody, Groups acting on graphs, Cambridge U.P. (more information than we need about trees).
D.L. Johnson, Presentations of groups, Cambridge University Press 1990, ISBN 0521378249, chapters 8 and 9 (useful for presentations and coset enumeration).

Sources for GAP and computational group theory

A book which shows great promise is
D.F. Holt, B. Eick, E.A. O'Brien, Handbook of computational group theory, Chapman &

Hall/CRC, c2005 (online access from the library eISBN 978-1-4200-3521-6)
This has the advantage that you can download it to your computer via access to the university library. It presents both general theory and also algorithms and their theory, although sometimes in a slightly short form.
The book by Johnson listed above is good for coset enumeration.
Information about GAP is obtained from its web site http://www.gap-system.org/
From the GAP site you can download GAP free of charge to your own computer. You may wish to explore the site, clicking on 'teaching', 'learning' and 'examples' (as a suggestion).
I mention also:
C.C. Sims, Computation with finitely presented groups, Cambridge University Press 1994, ISBN 0521432138

Course Assessment
I am not completely sure what to do about this because of the number of people enrolled. I will assign a set of homework problems roughly every 2 weeks, giving a total of six homework assignments altogether. My basic plan is that if you make a genuine attempt at 50% or more of the questions you will get an A for the course. You do not have to obtain correct solutions to these questions, only make genuine attempts (in my opinion). I believe that it is extremely difficult to obtain a sound and permanently lasting command of the material presented without doing some work which actively involves the student. I realize also that some people will already be very familiar with some things I teach, such as the use of GAP. In such a situation you may be able to talk me out of requiring you to hand in homework on that topic. As it is, it should be possible for everyone who wishes to obtain an A on this course.

Expectations of written work
Most of the time in the conventional homework problems, to satisfy my criterion of making a genuine attempt you will need to write down explanations for the calculations and arguments you make. Where explanations need to be given, these should be written out in sentences i.e. with verbs, capital letters at the beginning, periods at the end, etc. and not in an abbreviated form.
Some of the homework will be computer exercises in GAP. An essential part of what you hand in for these exercises will be a transcript of a GAP session, but it will help if you insert explanatory comments. Your work will exist as a computer file, and it would be possible to send it to me by email, but I do prefer to see a hard copy of what you have done.
I encourage you to form study groups. However everything to be handed in must be written up in your own words. If two students hand in identical assignments, they will both receive no credit.

Prerequisites
The content of the Math 8200 algebra sequence is sufficient as a prerequisite. As far as group theory is concerned, the topics I expect you to know include:
Lagrange's theorem, the isomorphism theorems, direct products of groups, properties of permutations (they may be written as products of disjoint cycles, conjugacy classes in the symmetric groups, the sign), structure of finitely-generated abelian groups, Jordan-Hölder theorem). Sylow's theorems.
For groups acting on trees it will be helpful to have some grasp of topology, but I am conscious that some members of the class may not have this! I will take this into account.

Incompletes
These will only be given in exceptional circumstances. A student must have satisfactorily completed all but a small portion of the work in the course, have a compelling reason for the incomplete, and must make prior arrangements with me for how the incomplete will be removed, well before the end of the quarter.

Date of this version of the schedule: 9/6/2010