

# AIMS Exercise Set # 1

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1. Determine the form of the single precision floating point arithmetic used in the computers at AIMS. What is the largest number that can be accurately represented? What is the smallest positive number  $n_1$ ? The second smallest positive number  $n_2$ ? Which is larger: the gap between  $n_1$  and 0 or the gap between  $n_1$  and  $n_2$ ? Discuss.

2. Determine the value of each of the following quantities using 4 digit rounding and four digit chopping arithmetic. Find the absolute and relative errors of your approximation. (a)  $\pi + e - \cos 22^\circ$ , (b)  $\frac{e^\pi - \pi^e}{\log \frac{10}{11}}$ .

3. (a) To how many significant decimal digits do the numbers  $\sqrt{10002}$  and  $\sqrt{10001}$  agree? (b) Subtract the two numbers. How many significant decimal digits are lost in the computation? (c) How might you rearrange the computation to obtain a more accurate answer.

4. (a) Verify that  $f(x) = 1 - \sin x$  and  $g(x) = \frac{\cos^2 x}{1 + \sin x}$  are identical functions.  
(b) Which function should be used for computations when  $x$  is near  $\frac{1}{2}\pi$ ? Why?  
(c) Which function should be used for computations when  $x$  is near  $\frac{3}{2}\pi$ ? Why?

## 5. Horner's Method

(a) Suppose  $x$  is a real number and  $n$  a positive integer. How many multiplications are needed to efficiently compute  $x^n$ ? *Hint:* The answer is not  $n - 1$ .  
(b) Verify the polynomial identity

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = a_0 + x(a_1 + x(a_2 + x(\cdots + xa_n)\cdots)).$$

Explain why the right hand side is to be preferred when computing the values of the polynomial  $p(x)$ .

6. Let

$$f(x) = e^x - \cos x - x.$$

- (a) Using calculus, what should the graph of  $f(x)$  look like for  $x$  near 0?
- (b) Using both single and double precision arithmetic, graph  $f(x)$  for  $|x| \leq 5 \times 10^{-8}$  and discuss what you observe.
- (c) How might you obtain a more realistic graph?

7. Consider the linear system of equations

$$1.1x + 2.1y = a, \quad 2x + 3.8y = b.$$

Solve the system for the following right hand sides: (i)  $a = 3.2$ ,  $b = 5.8$ ; (ii)  $a = 3.21$ ,  $b = 5.79$ ; (iii)  $a = 3.1$ ,  $b = 5.7$ . Discuss the conditioning of this system of equations.