Applied Linear Algebra, Second Edition

by Peter J. Olver and Chehrzad Shakiban

Corrections to First Printing (2018)

Last updated: May 6, 2024

⋆⋆⋆ Page 7 ⋆⋆⋆ line −6:

Change "i.e., $\mathrm{O}_{1\times n}$" to "i.e., $\mathrm{O}_{m\times 1}$, where its size m will, almost always, be fixed by the context."

⋆⋆⋆ Page 10 ⋆⋆⋆ Exercise 1.2.29 (a):

Change "i^{th} entry" to "j^{th} entry" and "j^{th} row" to "j^{th} column"

⋆⋆⋆ Page 14 ⋆⋆⋆ Back Substitution Pseudocode:

Correct summation limits: $x_i = \dfrac{1}{u_{ii}}\left(c_i - \sum_{j=i+1}^{n} u_{ij} x_j\right)$

⋆⋆⋆ Page 27 ⋆⋆⋆ Exercise 1.4.17 (a):

Change $(\pi(j), j)$ to $(j, \pi(j))$.

⋆⋆⋆ Page 35 ⋆⋆⋆ Exercise 1.5.15:

Change "everyy" to "every"

⋆⋆⋆ Page 44 ⋆⋆⋆ Exercise 1.6.12:

(*a*) Change "... $1 \times n$ column vector ..." to "... $n \times 1$ column vector ...".
(*b*) Change "... $1 \times m$ column vector ..." to "... $m \times 1$ column vector ..."

⋆⋆⋆ Page 44 ⋆⋆⋆ Exercise 1.7.14 (*b*):

Change lower right entry of matrix C_n from 1 to n:

$$C_n = \begin{pmatrix} 1 & -1 & & & & -1 \\ -1 & 2 & -1 & & & \\ & -1 & 3 & -1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & n-1 & -1 \\ -1 & & & & -1 & n \end{pmatrix}$$

⋆⋆⋆ Page 63 ⋆⋆⋆ line −16:

Change "... ofr basic variables ..." to "... of r basic variables ..."

⋆⋆⋆ Page 107 ⋆⋆⋆ line −6:

Correct the last entry in the second row of the second matrix: $\left(\begin{array}{ccc|c} 1 & 0 & -1 & b_1 \\ 0 & 1 & -2 & b_2 + b_1 \\ 0 & 0 & 0 & b_3 + 2b_2 + b_1 \end{array}\right)$

$\star\star\star$ Page 113 $\star\star\star$ line -9:

Change "… row vector with m zero entries." to "… row vector with n zero entries."

$\star\star\star$ Page 126 $\star\star\star$ line 3:

Change "Euler's formula (3.92) …" to "Euler's formula (2.49) …"

$\star\star\star$ Page 142 $\star\star\star$ Theorem 3.9:

Correct the final sentence to "Equality holds if and only if $\mathbf{v}$ and $\mathbf{w}$ are parallel vectors that point in the same direction, so $\langle\, \mathbf{v}\, , \mathbf{w}\, \rangle \geq 0$."

$\star\star\star$ Page 144 $\star\star\star$ Exercise 3.2.41 (d):

Change "… and $k \to \infty$." to "… as $k \to \infty$."

$\star\star\star$ Page 153 $\star\star\star$ Exercise 3.3.44:

Move Exercise 3.3.44 to the exercise set in following subsection since matrix norms are not introduced until there.

$\star\star\star$ Page 157 $\star\star\star$ Equation (3.53):

Make second 0 bold face:

$$q(\mathbf{x}) > 0 \qquad \text{for all} \qquad \mathbf{0} \neq \mathbf{x} \in \mathbb{R}^n. \tag{3.53}$$

$\star\star\star$ Page 159 $\star\star\star$ Equation (3.57):

Change $K = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ to $K = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$\star\star\star$ Page 168 $\star\star\star$ Example 3.45:

Change $(3,3)$ entry of K and coefficient of $q(\mathbf{x})$ from 8 to 11:

$$K = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 7 \\ 3 & 7 & 11 \end{pmatrix}, \qquad q(\mathbf{x}) = x_1^2 + 4\,x_1 x_2 + 6\,x_1 x_3 + 3\,x_2^2 + 14\,x_2 x_3 + 11\,x_3^2.$$

$\star\star\star$ Page 169 $\star\star\star$ line 20:

Change "… applying the the first phase of …" to "… applying the first phase of …"

$\star\star\star$ Page 176 $\star\star\star$ Exercise 3.6.17:

Change $\cos\theta - \cos\varphi$ to $\cos\theta + \cos\varphi$

$\star\star\star$ Page 178 $\star\star\star$ line 7:

Change "… their scalar cross product …" to "… minus their scalar cross product …"

$\star\star\star$ Page 185 $\star\star\star$ Last line of Theorem 4.5:

Change "… form a orthogonal basis for V." to "… form an orthogonal basis for V."

$\star\star\star$ Page 188 $\star\star\star$ last displayed equation:

Change the summation index on the first sum from j to i:

$$\|\mathbf{v}\|^2 = \langle\, \mathbf{v}\,,\mathbf{v}\,\rangle = \left\langle\, \sum_{i=1}^{n} c_i\,\mathbf{u}_i\,,\ \sum_{j=1}^{n} c_j\,\mathbf{u}_j \,\right\rangle = \sum_{i,j=1}^{n} c_i\,c_j\,\langle\, \mathbf{u}_i\,,\mathbf{u}_j\,\rangle = \sum_{i=1}^{n} c_i^2,$$

$\star\star\star$ Page 204 $\star\star\star$ Exercise 4.3.12. Change wording to:

Prove that every orthogonal upper triangular matrix is diagonal. What diagonal entries are possible?

$\star\star\star$ Page 205 $\star\star\star$ line -1:

Change "... entries of are positive." to "... entries are positive."

$\star\star\star$ Page 207 $\star\star\star$ line 5:

Change "For $j = 1, 2, 3,\ \ldots$" to "For $j = 2, 3, 4,\ \ldots$"

$\star\star\star$ Page 210 $\star\star\star$ line 9:

Change "... coincide with the first k columns of the eventual ..." to "... coincide with the first $k-1$ columns of the eventual ..."

$\star\star\star$ Page 210–211 $\star\star\star$ Example 4.29:

In the second displayed equation, change $\widehat{\mathbf{v}}_1$ to $\mathbf{w}_1$ and in the fourth displayed equation, change $\widehat{\mathbf{v}}_2$ to $\mathbf{w}_2$.

$\star\star\star$ Page 216 $\star\star\star$ Exercise 4.4.10:

Delete (e) and relabel parts (e,f,g,h) as (d,e,f,g).

In what is now part (d) change $\mathbf{v} \in \mathbb{R}^n$ to $\mathbf{v} \in \mathbb{R}^m$.

$\star\star\star$ Page 223 $\star\star\star$ line -11:

Change "... Proposition 4.41 ..." to "... Proposition 4.40 ..."

$\star\star\star$ Page 225 $\star\star\star$ line 9:

Change "... minimum value of $\sqrt{7}$ at $t = 1$." to "... minimum value of 7 at $t = 1$."

$\star\star\star$ Page 227 $\star\star\star$ Equation (4.54):

Change $\|\, q_6 \|^2$ to $\|\, q_5 \|^2$

$\star\star\star$ Page 230 $\star\star\star$

In the first displayed equation, delete the factor i in the first term on the second line:

$$\begin{aligned}\langle\, t^i\,, R_{j,k}\,\rangle &= \int_{-1}^{1} t^i\,R'_{j-1,k}(t)\,dt \\ &= t^i\,R_{j-1,k}(t)\,\Big|_{t=-1}^{1} - i\int_{-1}^{1} t^{i-1}\,R_{j-1,k}(t)\,dt = -\,i\,\langle\, t^{i-1}\,, R_{j-1,k}\,\rangle,\end{aligned}$$

$\star\star\star$ Page 244 $\star\star\star$ line 9:

Change $p(t\mathbf{y}) = a t^2 + 2 b t + c$ to $p(t\mathbf{y}) = a t^2 - 2 b t + c$

$\star\star\star$ Page 248 $\star\star\star$ line 3:

Add square root to right hand side of initial formula: $\|\mathbf{v}\| = \sqrt{v_1^2 + \frac{1}{2} v_2^2 + \frac{1}{3} v_3^2}$

$\star\star\star$ Page 251 $\star\star\star$ line 7:

Change "… hence is also a minimum." to "… hence $\mathbf{x}^\star + \mathbf{z}$ is also a minimum."

$\star\star\star$ Page 265 $\star\star\star$ line −6:

Change "An better strategy …" to "A better strategy …"

$\star\star\star$ Page 271 $\star\star\star$ Exercise 5.5.40 (c):

Change $x_1 = \frac{1}{3}(a+b),\ x_2 = \frac{2}{3}(a+b)$ to $x_1 = \frac{2}{3}a + \frac{1}{3}b,\ x_2 = \frac{1}{3}a + \frac{2}{3}b.$

$\star\star\star$ Page 271 $\star\star\star$ Exercise 5.5.40 (e):

Change $x_0 = \frac{1}{3}(a+b),\ x_1 = \frac{2}{3}(a+b)$ to $x_0 = \frac{2}{3}a + \frac{1}{3}b,\ x_1 = \frac{1}{3}a + \frac{2}{3}b.$

$\star\star\star$ Page 284 $\star\star\star$ Exercise 5.5.71 (d):

Change "Answer part (d) …" to "Answer part (c) …"

$\star\star\star$ Page 285 $\star\star\star$ line −14:

Change "… every 10–20 milliseconds …" to "… every 0.125 milliseconds …"

$\star\star\star$ Page 286 $\star\star\star$ Figure 5.16:

The three graphs in the second row should be reflected through the horizontal axis, i.e., reverse the sign of the function.

$\star\star\star$ Page 286 $\star\star\star$ line −14:

Change "… the imaginary parts, $\sin x$ and $-\sin 7x$." to
"… the imaginary parts, $-\sin x$ and $\sin 7x$."

$\star\star\star$ Page 296 $\star\star\star$ line 17:

Change "… signnificantly …" to "… significantly …"

$\star\star\star$ Page 301 $\star\star\star$ line 17:

Change "… partial differential equation that model …" to
"… partial differential equations that model …"

$\star\star\star$ Page 308 $\star\star\star$ Exercise 6.1.8 (b):

Change "Answer Exercise 6.1.8 when …" to "Answer part (a) when …"

$\star\star\star$ Page 309 $\star\star\star$ line 12:

Change "Moreover, since A …" to "Assuming at least one end of the chain is fixed, since A …"

$\star\star\star$ Page 310 $\star\star\star$ Exercise 6.1.16:

Change "Describe the mass–spring chains that gives rise to . . . " to "Describe mass–spring chains that give rise to . . . "

$\star\star\star$ Page 321 $\star\star\star$ line 2: Switch equation references:

". . . for electrical networks (6.28) and those of mass–spring chains (6.10)."

$\star\star\star$ Page 321 $\star\star\star$ Exercise 6.2.3:

Change ". . . Figure 6.5 . . . " to ". . . Exercise 6.2.2 . . . "

$\star\star\star$ Page 329 $\star\star\star$ line −3:

Change ". . . around the first node." to ". . . around the third node."

$\star\star\star$ Page 330 $\star\star\star$

Correct equation (6.57): $\mathbf{u} = \varepsilon\left(\frac{\sqrt{3}}{2}\,\mathbf{z}_1 - \frac{1}{2}\,\mathbf{z}_2 + \mathbf{z}_3\right) = \varepsilon\left(0,\ 0,\ \frac{\sqrt{3}}{2},\ \frac{1}{2},\ \frac{\sqrt{3}}{2},\ -\frac{1}{2}\right)^T$

and next to last displayed formula: $A^\star = \left(\begin{array}{cc|cc} -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$

and line immediately below: $\mathbf{z}_3^\star = \left(-\frac{\sqrt{3}}{2},\ \frac{1}{2},\ 0,\ 1\right)^T$

and final displayed formula: $A^{\star\star} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{pmatrix}$

$\star\star\star$ Page 331 $\star\star\star$ Correct first displayed formula:

$$K^{\star\star} = (A^{\star\star})^T A^{\star\star} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{2} \end{pmatrix}$$

$\star\star\star$ Page 350 $\star\star\star$ Equation (7.13):

Change $b_i = \ell[\mathbf{u}_i]$ to $b_i = \ell[\mathbf{v}_i]$

$\star\star\star$ Page 356 $\star\star\star$ line −10:

Change ". . . map is the choose a basis . . . " to ". . . map is to choose a basis . . . "

$\star\star\star$ Page 356 $\star\star\star$ last displayed equation:

Change initial v to boldface $\mathbf{v}$:

$$\begin{aligned} L[\mathbf{v}] = L[x_1\mathbf{v}_1 + \cdots + x_n\mathbf{v}_n] &= x_1 L[\mathbf{v}_1] + \cdots + x_n L[\mathbf{v}_n] \\ &= x_1\mathbf{e}_1 + \cdots + x_n\mathbf{e}_n = (x_1, x_2, \ldots, x_n)^T = \mathbf{x}, \end{aligned}$$

$\star\star\star$ Page 362 $\star\star\star$ line 6:

Change ". . . see (1.47) . . . " to ". . . see Proposition 1.25 . . . "

$\star\star\star$ Page 388–9 $\star\star\star$ Example 7.44:

Change the sign of second particular solution, so the corrected 4th, 6th, and 7th displayed formulas in the Example are

$$u_2^\star = \tfrac{1}{2}\, x \sin x.$$
$$u^\star = 3\, u_1^\star - 2\, u_2^\star = 3\, x - x \sin x.$$
$$u = 3\, x - x \sin x + c_1 \cos x + c_2 \sin x.$$

$\star\star\star$ Page 393 $\star\star\star$ Correct right hand side of the binomial formula in (7.78):

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k, \qquad \text{where} \qquad \binom{n}{k} = \frac{n\,!}{k\,!\,(n-k)\,!} \tag{7.78}$$

$\star\star\star$ Page 394 $\star\star\star$ Exercise 7.4.46:

Delete part (f).

$\star\star\star$ Page 396 $\star\star\star$ line -6:

Change $L[\,\mathbf{u}\,] = A\,\mathbf{v}$ to $L[\,\mathbf{u}\,] = A\,\mathbf{u}$

$\star\star\star$ Page 398 $\star\star\star$ Exercise 7.5.9:

Change $\mathbf{u} \in \mathbb{R}^n$ to $\mathbf{u} \in U$ and change $\langle\, \mathbf{f}\,, \mathbf{v}\,\rangle$ to $\langle\!\langle\, \mathbf{f}\,, \mathbf{v}\,\rangle\!\rangle$

$\star\star\star$ Page 399 $\star\star\star$ line 6:

Change $\langle\, L[\,\mathbf{u}\,]\,, L[\,\mathbf{u}\,]\,\rangle$ to $\langle\!\langle\, L[\,\mathbf{u}\,]\,, L[\,\mathbf{u}\,]\,\rangle\!\rangle$

$\star\star\star$ Page 418 $\star\star\star$ Exercise 8.2.32 (d):

Change "... converse to part (c) ..." to "... converse to part (b) ..."

$\star\star\star$ Page 422 $\star\star\star$ second displayed formula:

Correct first inequality:

$$r_i \geq |\, z - a_{ii} \,| \geq |\, a_{ii} \,| - |\, z \,| > r_i - |\, z \,|, \qquad \text{and hence} \qquad |\, z \,| > 0.$$

$\star\star\star$ Page 422 $\star\star\star$ Exercise 8.2.57:

This exercise relies on results in Section 8.5 and so should be moved there.

$\star\star\star$ Page 427 $\star\star\star$ Example 8.26, line 1:

Change "... Example 8.5 ..." to "... Example 8.6 ..."

$\star\star\star$ Page 431 $\star\star\star$ Exercise 8.4.1:

Change $W \subset \mathbb{R}^2$ to $W \subset \mathbb{R}^3$

$\star\star\star$ Page 442 $\star\star\star$ first displayed formula:

Delete second minus sign:

$$q\left(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right) = 2 \leq q(x,y) \leq 4 = q\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right) \qquad \text{for all} \quad x^2 + y^2 = 1.$$

⋆⋆⋆ Page 453 ⋆⋆⋆ Exercise 8.6.23:

Delete "(c)" immediately after "(b)", and relabel remaining parts as "(c), (d), (e)". In the resulting parts (d), (e), the formulas should be $p_A = (-1)^n m_A$.

⋆⋆⋆ Page 453 ⋆⋆⋆ Exercise 8.6.26 (c):

Change $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 3 & 3 & -4 \end{pmatrix}$ to $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ -3 & 3 & -4 \end{pmatrix}$

⋆⋆⋆ Page 453 ⋆⋆⋆ Exercise 8.6.27:

Insert period after "impractical"

⋆⋆⋆ Page 455 ⋆⋆⋆

There is a flaw in the proof of Theorem 8.63 as given. The argument in the text establishes the matrix equation

$$A Q = P \Sigma. \qquad (*)$$

If A has rank n, then Q is an $n \times n$ orthogonal matrix, and hence $Q Q^T = \mathrm{I}$. Thus, multiplying equation $(*)$ on the right by Q^T produces $A = A Q Q^T = P \Sigma Q^T$, which is the singular value decomposition (8.52). However, if the rank $r < n$, then Q is an $n \times r$ matrix with orthonormal columns, hence $Q^T Q = \mathrm{I}$, but it is *not* necessarily true that $Q Q^T = \mathrm{I}$, and so one cannot immediately establish the singular value decomposition (8.52) from $(*)$.

An alternative proof, that works in general, follows:

Proof: Let's begin by rewriting the desired factorization (8.52) as $A Q = P \Sigma$. The individual columns of this matrix equation are the vector equations

$$A \mathbf{q}_i = \sigma_i \mathbf{p}_i, \qquad \text{or, equivalently,} \qquad \mathbf{p}_i = \frac{A \mathbf{q}_i}{\sigma_i}, \qquad i = 1, \ldots, r, \tag{8.53}$$

relating the orthonormal columns of $Q = (\, \mathbf{q}_1 \; \mathbf{q}_2 \, \ldots \, \mathbf{q}_r \,)$ to the orthonormal columns of $P = (\, \mathbf{p}_1 \; \mathbf{p}_2 \, \ldots \, \mathbf{p}_r \,)$. Thus, our goal is to find vectors $\mathbf{p}_1, \ldots, \mathbf{p}_r$ and $\mathbf{q}_1, \ldots, \mathbf{q}_r$ that satisfy (8.53). To this end, we let $\mathbf{q}_1, \ldots, \mathbf{q}_n \in \mathbb{R}^n$ be the orthonormal eigenvector basis of the associated Gram matrix $A^T A$, where $\mathbf{q}_1, \ldots, \mathbf{q}_r$ are the singular eigenvectors, corresponding to the non-zero eigenvalues, i.e., the squares of the singular values, so

$$A^T A \mathbf{q}_i = \sigma_i^2 \mathbf{q}_i, \qquad i = 1, \ldots, r, \tag{8.54}$$

while $\mathbf{q}_{r+1}, \ldots, \mathbf{q}_n$ are the null eigenvectors, so

$$A \mathbf{q}_j = \mathbf{0}, \qquad A^T A \mathbf{q}_j = \mathbf{0}, \qquad j = r+1, \ldots, n, \tag{8.55}$$

where the first equation follows from the fact that A and $A^T A$ have the same kernel; see Proposition 8.37. Since $\mathbf{q}_1, \ldots, \mathbf{q}_n$ are orthonormal, for any $\mathbf{x} \in \mathbb{R}^n$, we have

$$\mathbf{x} = \sum_{i=1}^{n} (\mathbf{x} \cdot \mathbf{q}_i) \, \mathbf{q}_i = \sum_{i=1}^{n} (\mathbf{q}_i^T \mathbf{x}) \, \mathbf{q}_i, \qquad \text{and hence} \qquad A \mathbf{x} = \sum_{i=1}^{n} (\mathbf{q}_i^T \mathbf{x}) \, A \mathbf{q}_i = \left(\sum_{i=1}^{r} \sigma_i \mathbf{p}_i \mathbf{q}_i^T \right) \mathbf{x},$$

where we used (8.53), (8.55) in the final equality. Since this holds for all $\mathbf{x} \in \mathbb{R}^n$, we deduce that

$$A = \sum_{i=1}^{r} \sigma_i \mathbf{p}_i \mathbf{q}_i^T = P \Sigma Q^T,$$

where the final equality follows from Exercise 1.2.34 and the fact that Σ is diagonal. It remains to show that the vectors $\mathbf{p}_1, \ldots, \mathbf{p}_r$ are orthonormal; indeed, by the orthonormality of $\mathbf{q}_1, \ldots, \mathbf{q}_r$,

$$\mathbf{p}_i \cdot \mathbf{p}_j = \mathbf{p}_i^T \mathbf{p}_j = \frac{(A\,\mathbf{q}_i)^T A\,\mathbf{q}_j}{\sigma_i\,\sigma_j} = \frac{\mathbf{q}_i^T A^T A\,\mathbf{q}_j}{\sigma_i\,\sigma_j} = \frac{\sigma_j^2\,\mathbf{q}_i^T \mathbf{q}_j}{\sigma_i\,\sigma_j} = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases} \qquad Q.E.D.$$

$\Longrightarrow$ Thanks to Pasha Pylyavskyy for pointing this out, and Jeff Calder for supplying the above proof.

⋆⋆⋆ Page 456 ⋆⋆⋆ Equation (8.56):

Missing ^{T} on the final factor:

$$A^T = Q\,\Sigma\,P^T, \tag{8.56}$$

⋆⋆⋆ Page 456 ⋆⋆⋆ Correct equation (8.58):

$$Q^T K\,Q = Q^T A^T A\,Q = Q^T (P\,\Sigma\,Q^T)^T (P\,\Sigma\,Q^T)\,Q = Q^T Q\,\Sigma\,P^T P\,\Sigma\,Q^T Q = \Sigma^2, \tag{8.58}$$

$\Longrightarrow$ Thanks to Larry Baker for the correction.

⋆⋆⋆ Page 458 ⋆⋆⋆ displayed formula after (8.61):

Change first Σ^{-2} to Σ^2:

$$(A^T A)^{-1} A^T = (Q\,\Sigma^2\,Q^T)^{-1} (P\,\Sigma\,Q^T)^T = (Q\,\Sigma^{-2}\,Q^T)\,(Q\,\Sigma\,P^T) = Q\,\Sigma^{-1} P^T = A^+. \qquad Q.E.D.$$

⋆⋆⋆ Page 460 ⋆⋆⋆ :

Add 2 subscript to norms in third and fourth displayed equations:

$$\|\,\mathbf{u}\,\|_2 = \sqrt{c_1^2 + \;\cdots\; + c_n^2}\,, \qquad\qquad \|\,A\,\mathbf{u}\,\|_2 = \sqrt{c_1^2\,\sigma_1^2 + \;\cdots\; + c_r^2\,\sigma_r^2}\,.$$

⋆⋆⋆ Page 463 ⋆⋆⋆ Example 8.57:

Delete 0. from the list of singular values in the second line after (8.62). Also change the second and third sentences after the next displayed equation to:

"The singular values are their nonzero square roots. Note that these are fairly close to the eigenvalues of the original connected graph."

⋆⋆⋆ Page 465 ⋆⋆⋆ Exercise 8.7.20:

Insert space after n in "… $\operatorname{rank} B = n$ such that the Euclidean matrix …"

⋆⋆⋆ Page 472 ⋆⋆⋆ Equation (8.78):

Delete second ν in first term: $\nu\left(\sigma_1^2 + \;\cdots\; + \sigma_k^2\right) = \ldots$

$\star\star\star$ Page 473 $\star\star\star$ Exercise 8.8.1 (e):

Change , ., to , .2,: .9, −.4, −.8, .2, 1., −1.6, −1.2, −.7

$\star\star\star$ Page 474 $\star\star\star$ Exercise 8.8.12:

Change $\displaystyle\sum_{i=1}^{m} \operatorname{dist}(\mathbf{x}_i, L)$ to $\displaystyle\sum_{i=1}^{m} \operatorname{dist}(\mathbf{x}_i, L)^2$

$\star\star\star$ Page 489 $\star\star\star$ displayed formula after (9.24):

Change second bold face $\mathbf{0}$ to regular 0:

$$\mathbf{u}^{(k)} \to \mathbf{0} \qquad \text{if and only if} \qquad \| \mathbf{u}^{(k)} \| \to 0 \qquad \text{as} \qquad k \to \infty.$$

$\star\star\star$ Page 490 $\star\star\star$ beginning of paragraph before Example 9.15:

Change "The inequality (9.25) ..." to "The inequality (9.26) ..."

$\star\star\star$ Page 497 $\star\star\star$ line −2:

Change "By definition, ..." to "By Theorem 9.14, ..."

$\star\star\star$ Page 503 $\star\star\star$ line before displayed formula:

Change $v_k < v_j$ to $v_k < v_l$ to avoid conflict with summation index in formula.

$\star\star\star$ Page 505 $\star\star\star$ Exercise 9.3.16:

Change "... symmetric transition matrix ..." to "... symmetric regular transition matrix ..."

$\star\star\star$ Page 505 $\star\star\star$ Exercise 9.3.19:

Change "... doubly stochastic transition matrix ..." to "... doubly stochastic regular transition matrix ..."

$\star\star\star$ Page 508 $\star\star\star$ second displayed formula:

Change both **a**'s to **c**:

$$\mathbf{e}^{(k+1)} = \mathbf{u}^{(k+1)} - \mathbf{u}^\star = (T\,\mathbf{u}^{(k)} + \mathbf{c}) - (T\,\mathbf{u}^\star + \mathbf{c}) = T(\mathbf{u}^{(k)} - \mathbf{u}^\star) = T\,\mathbf{e}^{(k)},$$

$\star\star\star$ Page 522 $\star\star\star$ Exercise 9.4.35 (c):

Insert missing ω in formula: $\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \omega\,(\omega\,L + D)^{-1}\mathbf{r}^{(k)}$

$\star\star\star$ Page 525 $\star\star\star$ Example 9.42, line 2:

Change $\mathbf{u}^{(k)} = (\,1, 0, 0\,)^T$ to $\mathbf{u}^{(0)} = (\,1, 0, 0\,)^T$

$\star\star\star$ Page 530 $\star\star\star$ last line:

Change R_k to R_{k-1} and U^{-1} to U (twice):

$$R_{k-1} = P_k\,P_{k-1}^{-1} = \left(T_k\,\Lambda^k\,U\right)\left(T_{k-1}\,\Lambda^{k-1}\,U\right)^{-1} = T_k\,\Lambda\,T_{k-1}^{-1}.$$

⋆⋆⋆ Page 530 ⋆⋆⋆ The proof of Lemma 9.45 should be corrected as follows:

The last remaining item is a proof of Lemma 9.45. We write

$$S = (\,\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_n\,), \qquad S_k = \left(\,\mathbf{u}_1^{(k)}\ \mathbf{u}_2^{(k)}\ \ldots\ \mathbf{u}_n^{(k)}\,\right),$$

in columnar form. Let $t_{ij}^{(k)}$ denote the entries of the positive upper triangular matrix T_k. The first column of the limiting equation $S_k T_k \to S$ reads $t_{11}^{(k)}\,\mathbf{u}_1^{(k)} \to \mathbf{u}_1$. Since both $\mathbf{u}_1^{(k)}$ and $\mathbf{u}_1$ are unit vectors, and $t_{11}^{(k)} > 0$, it follows that

$$\|\, t_{11}^{(k)}\,\mathbf{u}_1^{(k)}\,\| = t_{11}^{(k)} \longrightarrow \|\,\mathbf{u}_1\,\| = 1, \quad \text{and hence the first column} \quad \mathbf{u}_1^{(k)} \longrightarrow \mathbf{u}_1.$$

The second column reads

$$t_{12}^{(k)}\,\mathbf{u}_1^{(k)} + t_{22}^{(k)}\,\mathbf{u}_2^{(k)} \longrightarrow \mathbf{u}_2.$$

Taking the inner product with $\mathbf{u}_1^{(k)} \to \mathbf{u}_1$ and using orthonormality, we deduce $t_{12}^{(k)} \to 0$, and hence $t_{22}^{(k)}\,\mathbf{u}_2^{(k)} \to \mathbf{u}_2$, which, by the previous reasoning, implies that $t_{22}^{(k)} \to 1$ and $\mathbf{u}_2^{(k)} \to \mathbf{u}_2$. The proof is completed by working through the remaining columns, using a similar argument at each step. The remaining details are left to the interested reader.

⟹ Thanks to Thomas Higham for the preceding two corrections.

⋆⋆⋆ Page 537 ⋆⋆⋆ second displayed formula:

Change $c_j + c_j^2$ to $c_{j-1} + c_j^2$:

$$\begin{aligned} A^{j+1}\mathbf{v} &= c_1 A\,\mathbf{v} + c_2 A^2\mathbf{v} + \ \cdots\ + c_{j-1}A^{j-1}\mathbf{v} + c_j A^j\,\mathbf{v} \\ &= c_j c_1 \mathbf{v} + (c_1 + c_j c_2)A\,\mathbf{v} + \ \cdots\ + (c_{j-2} + c_j c_{j-1})A^{j-2}\mathbf{v} + (c_{j-1} + c_j^2)A^{j-1}\mathbf{v} \in V^{(j)} \end{aligned}$$

⋆⋆⋆ Page 549 ⋆⋆⋆ Exercise 9.6.19 (*b*):

Change "... the solution ot the linear ..." to "... the solution to the linear ..."

⋆⋆⋆ Page 552 ⋆⋆⋆ Equation (9.133):

Change W_8 to W_2 here and also in the two following lines, for consistency with the revised Exercise 9.7.4; see below.

⋆⋆⋆ Page 553 ⋆⋆⋆ line before Example 9.56:

Change "... can be found [**18**, **88**]." to "... can be found in [**18**, **88**]."

⋆⋆⋆ Page 554 ⋆⋆⋆ Exercise 9.7.1 (a):

Change "... coefficients $c_{j,k}$." to
"... coefficients c_0 and $c_{j,k}$ for $j = 0, \ldots, 3$ and $k = 0, \ldots, 2^j - 1$."

⋆⋆⋆ Page 554 ⋆⋆⋆ Replace Exercise 9.7.4 by the following:

♡ 9.7.4. Let $f_n(x)$ denote the order n truncation of the wavelet expansion (9.136), and let $\mathbf{c} = (c_0, \ldots c_{j,k} \ldots) \in \mathbb{R}^{2^{n+1}}$ for $0 \le j \le n$, $0 \le k \le 2^j - 1$, denote its wavelet coefficients. (*a*) Explain why $f_n(x)$ is constant on each interval $x \in I_{i,n} = \big((i-1)\,2^{-n-1}, i\,2^{-n-1}\big)$ of length 2^{-n-1}, for $i = 1, \ldots, 2^{n+1}$. Let f_i denote its sample value thereon. (*b*) Explain why the truncated wavelet expansion defines a linear transformation that takes the coefficient vector $\mathbf{c}$ to the corresponding sample vector $\mathbf{f} = (f_1, f_2, \ldots, f_{2^{n+1}})^T$. ($c$) According to Theorem 7.5, the wavelet transformation must be given by matrix multiplication, $\mathbf{f} = W_n\,\mathbf{c}$, by a $2^{n+1} \times 2^{n+1}$ matrix W_n. Construct W_2, W_3, and W_4. (d) Prove that the columns of W_n are obtained as the values of the wavelet basis functions on the sample intervals. (e) Prove that the columns of W_n are orthogonal. (f) Is W_n an orthogonal matrix? (g) Find a formula for W_n^{-1}. (h) Explain why the order n wavelet transform is given by the inverse linear transformation: $\mathbf{c} = W_n^{-1}\,\mathbf{f}$.

⋆⋆⋆ Page 557 ⋆⋆⋆ Change summation in Equation (9.150):

$$\sum_k c_{2m+k}\, c_k = \begin{cases} 2, & m = 0, \\ 0, & m \neq 0, \end{cases} \tag{9.150}$$

⋆⋆⋆ Page 558 ⋆⋆⋆ Replace the final paragraph by the following:

Before explaining how to solve the Daubechies dilation equation, let us complete our discussion of orthogonality. It is easy to see that, by translation invariance of the inner product integral, since $\varphi(x)$ and $\varphi(x-m)$ are orthogonal whenever $m \neq 0$, so are $\varphi(x-k)$ and $\varphi(x-l)$ for all $k \neq l$. Next we seek to establish orthogonality of $\varphi(x-m)$ and $w(x)$. Combining the dilation equation (9.138) and the definition (9.142) of w, and then using (9.147, 148), produces

$$\begin{aligned} \langle\, w(x)\,, \varphi(x-m)\,\rangle &= \left\langle\, \sum_{j=0}^{p} (-1)^j c_{p-j}\, \varphi(2x-j)\,,\, \sum_{k=0}^{p} c_k\, \varphi(2x-2m-k) \,\right\rangle \\ &= \sum_{j,k=0}^{p} (-1)^j\, c_{p-j}\, c_k\, \langle\, \varphi(2x-j)\,, \varphi(2x-2m-k)\,\rangle \\ &= \sum_{j,k=0}^{p} (-1)^j\, c_{p-j}\, c_k\, \langle\, \varphi(x)\,, \varphi(x+j-2m-k)\,\rangle = \frac{1}{2} \sum_k (-1)^k\, c_{p-2m-k}\, c_k\, \|\,\varphi\,\|^2, \end{aligned}$$

where the sum is over all $0 \leq k \leq p$ such that $0 \leq 2m+k \leq p$. Now, if $p = 2q+1$ is odd, then each term in the final summation appears twice, with opposite signs, and hence the result is always zero — no matter what the coefficients $c_0, \ldots, c_p$ are! On the other hand, if $p = 2q$ is even, then it can be shown that orthogonality requires all $c_0 = \cdots = c_p = 0$, and hence $\varphi(x) \equiv 0$ is completely trivial and not of interest. Indeed, the particular cases $m = \pm q$ require $c_0 = c_p = 0$; with this, setting $m = \pm(q-1)$ requires $c_1 = c_{p-1} = 0$, and so on. Thus, to ensure orthogonality of the wavelet basis, the dilation equation (9.138) necessarily has an even number of terms, meaning that p must be an odd integer, as it is in the Haar and Daubechies versions (but not for the hat function). The proof of orthogonality of the translates $w(x-m)$ of the mother wavelet, along with all her wavelet descendants $w(2^j x - k)$, relies on a similar argument, and the details are left as Exercise 9.7.17.

⋆⋆⋆ Page 563 ⋆⋆⋆ Exercise 9.7.11:

Change $\varphi(x) = f(\log_2 x)/x$ to $\varphi(x) = x\, f(\log_2 x)$

⋆⋆⋆ Page 563 ⋆⋆⋆ Exercise 9.7.16 (b):

Change $2^{-j}\,\|\,\varphi\,\|$ to $2^{-j/2}\,\|\,\varphi\,\|$

$\star\star\star$ Page 563 $\star\star\star$ Exercise 9.7.22:

Change “Daubechies scaling equation” to “Daubechies dilation equation”.
Also change $i \geq p$ to $i \geq 3$

$\star\star\star$ Page 579 $\star\star\star$ lines $-6, -5$:

Change

“An equilibrium point is called *globally stable* if the stability condition holds for *all* $\varepsilon > 0$.”

to

“An equilibrium point is called *globally stable* if it is locally stable and, in addition, every solution remains bounded for all $t \geq t_0$.”

$\star\star\star$ Page 585 $\star\star\star$

Delete Exercise 10.2.23, which repeats Exercise 10.2.21.

$\star\star\star$ Page 587 $\star\star\star$ line 3:

Delete the superfluous condition $\Delta > 0$ which is a consequence of $\det A < 0$.

$\star\star\star$ Page 594 $\star\star\star$ third displayed formula:

Change $U(0)$ to $U(t_0)$: $U(t_0) = e^{0\,A} B = \mathrm{I}\, B = B$

$\star\star\star$ Page 598 $\star\star\star$ Exercise 10.4.26:

Change part (b) to

(b) *True or false*: Do the eigenvalues have the same multiplicities?

and delete the Hint.

$\star\star\star$ Page 599 $\star\star\star$ line before (10.49):

change (10.39) to (10.47)

Acknowledgements:

Many thanks to Lawrence Baker, Jeff Calder, Jefferson Carpenter, David Foster, Samuel Hehir, Thomas Higham, Connor Kinney, James Meiss, Nathan Mihm, Marc Paolella, Pasha Pylyavskyy, and Alexander Voronov for sending us their corrections to the printed text and the solutions manuals.