Applied Linear Algebra

by Peter J. Olver and Chehrzad Shakiban

Corrections to Instructor's Solution Manual

Last updated: December 15, 2013

1.2.4 (*d*) $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \ \mathbf{x} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix};$

(*e*) $A = \begin{pmatrix} 5 & 3 & -1 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix},$

(*f*) $\mathbf{b} = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}.$

1.4.15 (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$

1.8.4 (*i*) $a \neq b$ and $b \neq 0$; (*ii*) $a = b \neq 0$, or $a = -2, \ b = 0$; (*iii*) $a \neq -2, \ b = 0$.

1.8.23 (*e*) $(0, 0, 0)^T$;

2.2.28 (a) By induction, we can show that, for $n \geq 1$ and $x > 0$,

$$f^{(n)}(x) = \frac{Q_{n-1}(x)}{x^{2n}} \, e^{-1/x^2},$$

where $Q_{n-1}(x)$ is a polynomial of degree $n-1$. Thus,

$$\lim_{x \to 0^+} f^{(n)}(x) = \lim_{x \to 0^+} \frac{Q_{n-1}(x)}{x^{2n}} \, e^{-1/x^2} = Q_{n-1}(0) \lim_{y \to \infty} y^{2n} \, e^{-y} = 0 = \lim_{x \to 0^-} f^{(n)}(x),$$

because the exponential e^{-y} goes to zero faster than any power of y goes to ∞.

2.5.5 (*b*) $\mathbf{x}^\star = (1, -1, 0)^T, \quad \mathbf{z} = z\left(-\frac{2}{7}, -\frac{1}{7}, 1\right)^T;$

2.5.31 (*d*) ... while $\operatorname{coker} U$ has basis $(0, 0, 0, 1)^T$.

2.5.32 (b) Yes, the preceding example can put into row echelon form by the following elementary row operations of type #1:

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \xrightarrow{R_1 \mapsto R_1 + R_2} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \xrightarrow{R_2 \mapsto R_2 - R_1} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

Indeed, Exercise 1.4.18 shows how to interchange any two rows, modulo multiplying one by an inessential minus sign, using only elementary row operations of type #1. As a consequence, one can reduce *any* matrix to row echelon form without any row interchanges!

(The answer in the manual implicitly assumes that the row operations had to be done in the standard order. But this is not stated in the exercise as written.)

2.5.42 True. If $\ker A = \ker B \subset \mathbb{R}^n$, then both matrices have n columns, and so $n - \operatorname{rank} A = \dim \ker A = \dim \ker B = n - \operatorname{rank} B$.

3.1.6 (b) ... plane has length ...

3.1.10 (c) If $\mathbf{v}$ is any element of V, then we can write $\mathbf{v} = c_1 \mathbf{v}_1 + \cdots + c_n \mathbf{v}_n$ as a linear combination of the basis elements, and so, by bilinearity,

$$\begin{aligned} \langle \mathbf{x} - \mathbf{y}, \mathbf{v} \rangle &= c_1 \langle \mathbf{x} - \mathbf{y}, \mathbf{v}_1 \rangle + \cdots + c_n \langle \mathbf{x} - \mathbf{y}, \mathbf{v}_n \rangle \\ &= c_1 \big(\langle \mathbf{x}, \mathbf{v}_1 \rangle - \langle \mathbf{y}, \mathbf{v}_1 \rangle \big) + \cdots + c_n \big(\langle \mathbf{x}, \mathbf{v}_n \rangle - \langle \mathbf{y}, \mathbf{v}_n \rangle \big) = 0. \end{aligned}$$

Since this holds for all $\mathbf{v} \in V$, the result in part (a) implies $\mathbf{x} = \mathbf{y}$.

3.2.11 (b) Missing square on $\langle \mathbf{v}, \mathbf{w} \rangle$ in formula:

$$\sin^2 \theta = 1 - \cos^2 \theta = \frac{\| \mathbf{v} \|^2 \, \| \mathbf{w} \|^2 - \langle \mathbf{v}, \mathbf{w} \rangle^2}{\| \mathbf{v} \|^2 \, \| \mathbf{w} \|^2} = \frac{(\mathbf{v} \times \mathbf{w})^2}{\| \mathbf{v} \|^2 \, \| \mathbf{w} \|^2}.$$

3.4.22 (ii) and (v) Change "null vectors" to "null directions".

3.4.33 (a) $L = (A^T)^T A^T$ is ...

3.5.3 (b) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}.$

3.6.33 Change all $\mathbf{w}$'s to $\overline{\mathbf{v}}$'s.

4.2.4 (c) When $|\, b \,| \geq 2$, the minimum is ...

4.4.23 Delete "(c)". (Just the label, not the formula coming afterwards.)

4.4.27 (a) Change "the interpolating polynomial" to "an interpolating polynomial".

4.4.52 The solution given in the manual is for the square $S = \{\, 0 \le x \le 1,\ 0 \le y \le 1 \,\}$. When $S = \{\, -1 \le x \le 1,\ -1 \le y \le 1 \,\}$, use the following:

Solution: (a) $z = \frac{2}{3}$, (*b*) $z = \frac{3}{5}(x - y)$, (*c*) $z = 0$.

5.1.14 One way to solve this is by direct computation. A more sophisticated approach is to apply the Cholesky factorization (3.70) to the inner product matrix: $K = M M^T$. Then, $\langle \mathbf{v}, \mathbf{w} \rangle = \mathbf{v}^T K \mathbf{w} = \widehat{\mathbf{v}}^T \widehat{\mathbf{w}}$ where $\widehat{\mathbf{v}} = M^T \mathbf{v}$, $\widehat{\mathbf{w}} = M^T \mathbf{w}$. Therefore, $\mathbf{v}_1, \mathbf{v}_2$ form an orthonormal basis relative to $\langle \mathbf{v}, \mathbf{w} \rangle = \mathbf{v}^T K \mathbf{w}$ if and only if $\widehat{\mathbf{v}}_1 = M^T \mathbf{v}_1$, $\widehat{\mathbf{v}}_2 = M^T \mathbf{v}_2$, form an orthonormal basis for the dot product, and hence of the form determined in Exercise 5.1.11. Using this we find: (a) $M = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{2} \end{pmatrix}$, so $\mathbf{v}_1 = \begin{pmatrix} \cos\theta \\ \frac{1}{\sqrt{2}} \sin\theta \end{pmatrix}$, $\mathbf{v}_2 = \pm \begin{pmatrix} -\sin\theta \\ \frac{1}{\sqrt{2}} \cos\theta \end{pmatrix}$, for any $0 \le \theta < 2\pi$. (*b*) $M = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, so $\mathbf{v}_1 = \begin{pmatrix} \cos\theta + \sin\theta \\ \sin\theta \end{pmatrix}$, $\mathbf{v}_2 = \pm \begin{pmatrix} \cos\theta - \sin\theta \\ \cos\theta \end{pmatrix}$, for any $0 \le \theta < 2\pi$.

5.4.15 $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = x^2 - \frac{1}{3}$, $p_4(x) = x^3 - \frac{9}{10}x$.
(The solution given is for the interval $[0,1]$, not $[-1,1]$.)

5.5.6 (*ii*) (*c*) $\begin{pmatrix} \frac{23}{43} \\ \frac{19}{43} \\ -\frac{1}{43} \end{pmatrix} \approx \begin{pmatrix} .5349 \\ .4419 \\ -.0233 \end{pmatrix}$,

5.5.6 (*ii*) (*d*) $\begin{pmatrix} \frac{614}{26883} \\ -\frac{163}{927} \\ \frac{1876}{8961} \end{pmatrix} \approx \begin{pmatrix} .0228 \\ -.1758 \\ .2094 \end{pmatrix}$.

5.6.20 (*c*) The solution corresponds to the revised exercise for the system

$$x_1 + 2x_2 + 3x_3 = b_1,\ \ x_2 + 2x_3 = b_2,\ \ 3x_1 + 5x_2 + 7x_3 = b_3,\ \ -2x_1 + x_2 + 4x_3 = b_4.$$

For the given system, the cokernel basis is $(-3, 1, 1, 0)^T$, and the compatibility condition is $-3b_1 + b_2 + b_3 = 0$.

5.7.2 (a,b,c) To avoid any confusion, delete the superfluous last sample value in the first equation:

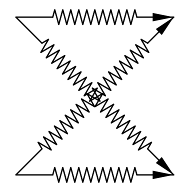
(a) (i) $f_0 = 2,\ f_1 = -1,\ f_2 = -1.$ (ii) $e^{-\,\mathrm{i}\,x} + e^{\,\mathrm{i}\,x} = 2\cos x;$

(b) (i) $f_0 = 1,\ f_1 = 1 - \sqrt{5}\,,\ f_2 = 1 + \sqrt{5}\,,\ f_3 = 1 + \sqrt{5}\,,\ f_4 = 1 - \sqrt{5}\,;$
(ii) $e^{-2\,\mathrm{i}\,x} - e^{-\,\mathrm{i}\,x} + 1 - e^{\,\mathrm{i}\,x} + e^{2\,\mathrm{i}\,x} = 1 - 2\cos x + 2\cos 2x;$

(c) (i) $f_0 = 6,\ f_1 = 2 + 2e^{2\pi\,\mathrm{i}/5} + 2e^{-4\pi\,\mathrm{i}/5} = 1 + .7265\,\mathrm{i}\,,$
$f_2 = 2 + 2e^{2\pi\,\mathrm{i}/5} + 2e^{4\pi\,\mathrm{i}/5} = 1 + 3.0777\,\mathrm{i}\,,$
$f_3 = 2 + 2e^{-2\pi\,\mathrm{i}/5} + 2e^{-4\pi\,\mathrm{i}/5} = 1 - 3.0777\,\mathrm{i}\,,$
$f_4 = 2 + 2e^{-2\pi\,\mathrm{i}/5} + 2e^{4\pi\,\mathrm{i}/5} = 1 - .7265\,\mathrm{i}\,;$
(ii) $2e^{-2\,\mathrm{i}\,x} + 2 + 2e^{\,\mathrm{i}\,x} = 2 + 2\cos x + 2\,\mathrm{i}\,\sin x + 2\cos 2x - 2\,\mathrm{i}\,\sin 2x;$

(d) (i) $f_0 = f_1 = f_2 = f_4 = f_5 = 0,\ f_3 = 6;$ (ii) $1 - e^{\,\mathrm{i}\,x} + e^{2\,\mathrm{i}\,x} - e^{3\,\mathrm{i}\,x} + e^{4\,\mathrm{i}\,x} - e^{5\,\mathrm{i}\,x} = 1 - \cos x + \cos 2x - \cos 3x + \cos 4x - \cos 5x + \mathrm{i}\,(-\sin x + \sin 2x - \sin 3x + \sin 4x - \sin 5x).$

6.2.1 (b) The solution given in the manual corresponds to the revised exercise with incidence matrix $\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$. For the given matrix, the solution is



6.3.5 (b)

$$\begin{aligned} \tfrac{3}{2}u_1 - \tfrac{1}{2}v_1 - u_2 &= f_1, \\ -\tfrac{1}{2}u_1 + \tfrac{3}{2}v_1 &= g_1, \\ -u_1 + \tfrac{3}{2}u_2 + \tfrac{1}{2}v_2 &= f_2, \\ \tfrac{1}{2}u_2 + \tfrac{3}{2}v_2 &= g_2. \end{aligned}$$

7.4.13 (ii) (b) $v(t) = c_1 e^{2t} + c_2 e^{-t/2}$

7.4.19 Set $d = c$ in the written solution.

7.5.8 (d) *Note*: The solution is correct provided, for $L\colon V \to V$, one uses the same inner product on the domain and target copies of V. If different inner products are used, then the identity map is not self-adjoint, $\mathrm{I}^* \neq \mathrm{I}$, and so, in this more general situation, $(L^{-1})^* \neq (L^*)^{-1}$.

8.3.21 (a) $\begin{pmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{8}{3} & \frac{1}{3} \end{pmatrix},$

8.5.26 Interchange solutions (b) and (c).

9.4.38 Change $e^{t:A}$ to e^{tA}.

10.5.12 The solution given in the manual is for $\mathbf{b} = (-2,-1,7)^T$. When $\mathbf{b} = (4,0,4)^T$, use the following:

Solution:

(a) $\mathbf{x} = \begin{pmatrix} \frac{88}{69} \\ \frac{12}{23} \\ \frac{56}{69} \end{pmatrix} = \begin{pmatrix} 1.27536 \\ .52174 \\ .81159 \end{pmatrix}$;

(b) $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{x}^{(2)} = \begin{pmatrix} 1.50 \\ .50 \\ .75 \end{pmatrix}$, $\mathbf{x}^{(3)} = \begin{pmatrix} 1.2500 \\ .5675 \\ .7500 \end{pmatrix}$, with error $\mathbf{e}^{(3)} = \begin{pmatrix} -.02536 \\ .04076 \\ -.06159 \end{pmatrix}$;

(c) $\mathbf{x}^{(k+1)} = \begin{pmatrix} 0 & -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 0 \end{pmatrix} \mathbf{x}^{(k)} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$;

(d) $\mathbf{x}^{(1)} = \begin{pmatrix} 1.0000 \\ .2500 \\ .8125 \end{pmatrix}$, $\mathbf{x}^{(2)} = \begin{pmatrix} 1.34375 \\ .53906 \\ .79883 \end{pmatrix}$, $\mathbf{x}^{(3)} = \begin{pmatrix} 1.26465 \\ .51587 \\ .81281 \end{pmatrix}$; the error at the third iteration is $\mathbf{e}^{(3)} = \begin{pmatrix} -.01071 \\ -.00587 \\ .001211 \end{pmatrix}$; the Gauss-Seidel approximation is more accurate.

(e) $\mathbf{x}^{(k+1)} = \begin{pmatrix} 0 & -\frac{1}{4} & \frac{1}{2} \\ 0 & -\frac{1}{16} & \frac{3}{8} \\ 0 & \frac{3}{64} & -\frac{1}{32} \end{pmatrix} \mathbf{x}^{(k)} + \begin{pmatrix} 1 \\ \frac{1}{4} \\ \frac{13}{16} \end{pmatrix}$;

(f) $\rho(T_J) = \frac{\sqrt{3}}{4} = .433013$,

(g) $\rho(T_{GS}) = \frac{3+\sqrt{73}}{64} = .180375$, so Gauss–Seidel converges about $\log \rho_{GS} / \log \rho_J = 2.046$ times as fast.

(h) Approximately $\log(.5 \times 10^{-6}) / \log \rho_{GS} \approx 8.5$ iterations.

(i) Under Gauss–Seidel, $\mathbf{x}^{(9)} = \begin{pmatrix} 1.27536 \\ .52174 \\ .81159 \end{pmatrix}$, with error $\mathbf{e}^{(9)} = 10^{-6} \begin{pmatrix} -.3869 \\ -.1719 \\ .0536 \end{pmatrix}$.

11.1.11 Change upper integration limit in the formula for $a = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} \left(\int_0^y f(z)\, dz \right) dy$.

11.2.2 (f) $\varphi(x) = \frac{1}{2}\,\delta(x-1) - \frac{1}{5}\,\delta(x-2)$; $\displaystyle\int_a^b \varphi(x)\, u(x)\, dx = \frac{1}{2}u(1) - \frac{1}{5}u(2)$ for $a < 1 < 2 < b$.

11.2.8 (d) $f'(x) = 4\,\delta(x+2) + 4\,\delta(x-2) + \begin{cases} 1, & |\,x\,| > 2, \\ -1, & |\,x\,| < 2, \end{cases}$

$$= 4\,\delta(x+2) + 4\,\delta(x-2) + 1 - 2\,\sigma(x+2) + 2\,\sigma(x-2),$$

$$f''(x) = 4\,\delta'(x+2) + 4\,\delta'(x-2) - 2\,\delta(x+2) + 2\,\delta(x-2).$$

11.2.13 (a) The symbol on the vertical axis should be n, not $\frac{1}{2}\,n$.

11.2.15 Replace $-\,\sigma_y(x)$ by $\sigma_y(x) - 1$.

11.2.17 In displayed equation, delete extra parenthesis at end of $\delta_\xi^{(k)}(x)$ (twice); replace u by u' in penultimate term: $-\,\langle\, \delta_\xi^{(k)}\,, u'\,\rangle$.

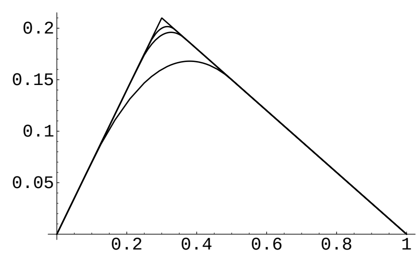
11.2.19 Delete one repetition of "On the other hand".

11.2.31 (a)

$$u_n(x) = \begin{cases} x\,(1-y), & 0 \le x \le y - \frac{1}{n}\,, \\ -\frac{1}{4}\,n\,x^2 + \left(\frac{1}{2}\,n - 1\right)x\,y - \frac{1}{4}\,n\,y^2 + \frac{1}{2}\,y + \frac{1}{2}\,x - \frac{1}{4\,n}, & |\,x-y\,| \le \frac{1}{n}\,, \\ y\,(1-x), & y + \frac{1}{n} \le x \le 1. \end{cases}$$

(b) Since $u_n(x) = G(x,y)$ for all $|\,x-y\,| \ge \frac{1}{n}$, we have $\lim\limits_{n\to\infty} u_n(x) = G(x,y)$ for all $x \ne y$, while $\lim\limits_{n\to\infty} u_n(y) = \lim\limits_{n\to\infty}\left(y - y^2 - \frac{1}{4\,n}\right) = y - y^2 = G(y,y)$. (Or one can appeal to continuity to infer this.) This limit reflects the fact that the external forces converge to the delta function: $\lim\limits_{n\to\infty} f_n(x) = \delta(x-y)$.

(c)



11.2.35 End of first line in final equation: $\dfrac{\partial I}{\partial x} + \dfrac{\partial I}{\partial z}\,\dfrac{d\alpha}{dx} + \dfrac{\partial I}{\partial w}\,\dfrac{d\beta}{dx}$

11.3.3 (*c*) (*i*) $u_\star(x) = \frac{1}{2}x^2 - \frac{5}{2} + x^{-1}$,

(*ii*) $\mathcal{P}[u] = \int_1^2 \left[\frac{1}{2}x^2 (u')^2 + 3x^2 u \right] dx, \qquad u'(1) = u(2) = 0$,

(*iii*) $\mathcal{P}[u_\star] = -\frac{37}{20} = -1.85$,

(*iv*) $\mathcal{P}[x^2 - 2x] = -\frac{11}{6} = -1.83333, \quad \mathcal{P}\left[-\sin\frac{1}{2}\pi x\right] = -1.84534$.

(*d*) (*iv*) $\mathcal{P}\left[-\frac{1}{4}(x+1)(x+2)\right] = -.0536458$,

$\mathcal{P}\left[\frac{3}{20}x(x+1)(x+2)\right] = -.0457634$.

11.3.5 (*c*) Unique minimizer: $u_\star(x) = \dfrac{x+1}{2} - \dfrac{1}{2}\log\dfrac{(1+\sin x)\cos 1}{(1-\sin 1)\cos x}$.
(The given solution is for the boundary conditions $u(-1) = u(1) = 0$.)

11.3.21 $u(x) = \dfrac{x^3 - 1}{9} + \dfrac{2\log x}{9\log 2}$

11.5.3 Change sign of last expression: $= -\dfrac{1}{\omega^2} + \dfrac{e^{-\omega/2}e^{\omega x} + e^{\omega}e^{-\omega x}}{\omega^2(1 + e^{\omega/2})}$.

11.5.7 (*b*) For $\lambda = -\omega^2 < 0$, $\quad G(x,y) = \begin{cases} \dfrac{\sinh\omega(y-1)\sinh\omega x}{\omega\sinh\omega}, & x < y, \\ \dfrac{\sinh\omega(x-1)\sinh\omega y}{\omega\sinh\omega}, & x > y; \end{cases}$

for $\lambda = 0$, $\quad G(x,y) = \begin{cases} x(y-1), & x < y, \\ y(x-1), & x > y; \end{cases}$

for $\lambda = \omega^2 \neq n^2\pi^2 > 0$, $\quad G(x,y) = \begin{cases} \dfrac{\sin\omega(y-1)\sin\omega x}{\omega\sin\omega}, & x < y, \\ \dfrac{\sin\omega(x-1)\sin\omega y}{\omega\sin\omega}, & x > y. \end{cases}$

11.5.9 (*c*) (*i*,*ii*,*iii*) Replace $\int_a^b$ by $\int_1^2$.

Acknowledgments: We would particularly like to thank to David Hiebeler, University of Maine, for alerting us to many of these corrections.

