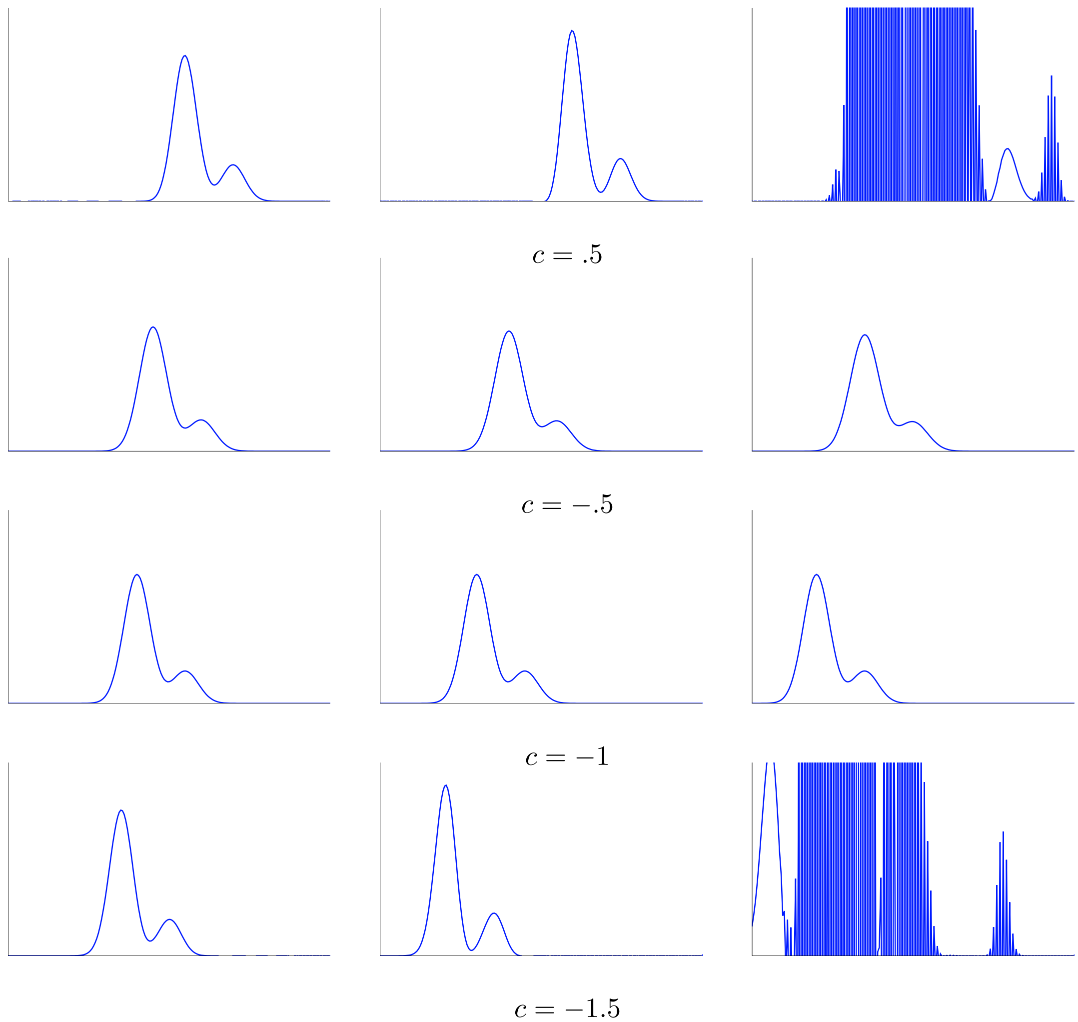

Figure 5.5. Numerical solutions to the transport equation.

solution when $c = -.5$ is a bit more reasonable, although one can already observe some degradation due to the relatively low accuracy of the scheme. This can be alleviated by employing a smaller step size. The case $c = -1$ looks exceptionally good, and you are asked to provide an explanation in Exercise 5.3.6.

The CFL Condition

There are two ways to understand the observed numerical instability. First, we recall that the exact solution (5.36) is constant along the characteristic lines $x = ct + \xi$, and hence the value of $u(t, x)$ depends only on the initial value $f(\xi)$ at the point $\xi = x - ct$. On the other hand, at time $t = t_j$, the numerical solution $u_{j,m} \approx u(t_j, x_m)$ computed using (5.38) depends on the values of $u_{j-1,m}$ and $u_{j-1,m+1}$. The latter two values have